# 非線性系統之可拓滑動模式控制

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## 摘要

本文提出一基於滑動模式控制架構之新式可拓控制器。所提控制器之控制策略主要分為兩部分：一是等效滑動模式控制器，其可由原始的受控系統直接推導得來。另一為眾所皆知的強健控制器，用於補償系統的不確定性及外部擾動項。整合可拓集合理論之重要概念及滑動模式控制，吾人可成功地實現了一強健可拓控制器。本研究首先架構了一個無須受控系統任何經驗知識之基本可拓控制器，接著選定一組可拓特性函數，用於主宰受控制系統的狀態變化，再將一組符合適應性控制策略的可調參數，併入上述可拓控制器，以達成強健控制的目的。如此系統亦稱衝擊控制，乃用於確保系統的強健控制性。實際上，所提之可拓控制器是滿足李雅普諾夫穩定性的。最後，以一個非線性系統之可拓滑動控制為例，來驗證所提方法之可行性。

**關鍵詞 :** 可拓集合、可拓控制、滑動模式控制、系統穩定度

# Extension Sliding Mode Control for Nonlinear Systems

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## Abstract

This paper presents a novel extension controller based on a scheme for sliding mode control. The control strategy of the proposed controller is classified in two parts. One is the equivalent control of sliding mode control, which can be directly obtained from the knowing nominal system. The other is the well-known robust control, which is used to compensate for the system uncertainty and external disturbance. Integrating the essential concept of extension set theory with the sliding mode control, we can easily realize the robust extension controller. In this study, we first organize a basic extension controller without any specific expert knowledge about the controlled system. According to the determined extension characteristic functions, the functions supervise system states' information under control. A set of parameters, which will be adapted using adaptation strategy, is incorporated into the extension controller such that a robust control term, also called a hitting control, is established to guarantee the system's stability. As a result, the proposed extension controller is stable in the sense of Lyapunov. Finally, we use the proposed extension controller to control a nonlinear system to verify its effectiveness and ability.

**Key Words :** Extension Set, Extension Control, Sliding Mode Control, System Stability.

# I. Introduction

Extension sets theory, originally proposed in 1983 by Prof. Cai in China, forms the basis for research on extension engineering methods, which have formed an important quantitative analysis methodology for physical applications [1]. Recently, various applications such as optimization, identification, pattern recognition, clustering, decision making and controller design have been presented in the literature [2-9]. However, generally speaking, extension control has not developed well. There are two main disadvantages to the control field if we employ the extension set to implement a control task. First, the concept of extension set is attractive, a good structure for the extension controller has not been developed so far. Unlike the fuzzy controller, which is specifically consists of control rules, membership functions and a mechanism to implement the approximation reasoning. Second, the valuation set in the characteristic function of extension set is bounded in $(-\infty,1]$, so the problem arises of what kind of strategy should be utilized to design the extension controller based on the extension sets we been defined. In [9], the authors used the linguistic control rules, which were the same as the fuzzy controller, to design the extension controller. In addition, heuristic control input according to the developed characteristic function was reported in [8]. From these papers, we can see that an important issue in designing the extension controller is to develop a systematic design approach for the extension controller.

Sliding mode controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision [10-13]. Generally speaking, the system states are driven toward a user-defined sliding surface. If the state trajectory can be maintained on the surface, the overall control system is insensitive to uncertainty and disturbance. In this paper, a scheme of sliding mode control is incorporated into the extension controller design, such that the control system resulting from the extension control has the same merits as the sliding mode control.

This paper is organized as follows. Section 2 provides a brief description of the sliding mode control system. In Section 3, the extension controller structure is introduced; and the design of an extension controller based on the sliding mode control is then derived. Section 4 provides a simulation example for a simple nonlinear time-varing system, which is used to illustrate the performance of the proposed controller. Finally, conclusions are given in Section 5.

# II. System description

In this section, the sliding mode control system is first introduced, and then the structure of the extension controller based on the sliding mode is constructed. Consider the following $n$th-order dynamical systems

$$x^{(n)} = f(\mathbf{x}) + g(\mathbf{x})u \tag{1}$$

where $u \in R$ is control input; $f$ and $g$ are unknown nonlinear continuous functions, but with bounds known to be $|f| \le F$ and $0 < \underline{g} \le g \le \bar{g}$ ; $\mathbf{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T$ is the state vector. The control task is to provide a proper control input $u$ such that the error vector $\mathbf{e} = \mathbf{x} - \mathbf{x}_d$ can be minimized, where $\mathbf{x}_d = [x_d, \dot{x}_d, \cdots, x_d^{(n-1)}]^T$ is the desired state vector. In the sliding mode control, we first define the sliding surface

$$S = \left(\frac{d}{dt} + \lambda\right)^{(n-1)} \mathbf{e} = e^{(n-1)} + a_1 e^{(n-2)} + \cdots + a_{n-1} e \tag{2}$$

where $\lambda$ is a strictly positive constant.

In order to guarantee $S = 0$ to be reached, the control input $u$ should be chosen to satisfy the following sliding condition

$$S\dot{S} \le -\eta |S| \tag{3}$$

where $\eta$ is a positive constant. By (1) and (2), we have

$$\dot{S} = e^{(n)} + a_1 e^{(n-1)} + \cdots + a_{n-1}\dot{e} = f + gu - x_d^{(n)} + a_1 e^{(n-1)} + \cdots + a_{n-1}\dot{e} \tag{4}$$

Furthermore, define

$$\hat{g} = \sqrt{\bar{g}\,\underline{g}} \tag{5}$$

and

$$\breve{g} = \sqrt{\frac{\overline{g}}{\underline{g}}} \tag{6}$$

Since $0 < \underline{g} \le g \le \overline{g}$ , we have

$$\breve{g}^{-1} \le \frac{\hat{g}}{g} \le \breve{g} \tag{7}$$

The control input of the sliding mode controller consists of two components, one is the equivalent control $u_{eq}$ and the other is the hitting control $u_h$ . Let

$$u = u_{eq} + u_h = \hat{g}^{-1}\breve{u} - \hat{g}^{-1} G \operatorname{sgn}(S) \tag{8}$$

where

$$\breve{u} = x_d^{(n)} - a_1 e^{(n-1)} - \cdots - a_{n-1}\dot{e} \tag{9}$$

Inserting (8) into (4), we have

$$\dot{S} = f + (g\hat{g}^{-1} - 1)\breve{u} - g\hat{g}^{-1} G \operatorname{sgn}(S) \tag{10}$$

Then it is easy to find the following optimal *G* satisfying the sliding condition

$$G^* = \eta g^{-1}\hat{g} + \left| g^{-1}\hat{g} f + (1 - g^{-1}\hat{g})\breve{u} \right| \tag{11}$$

Since both function *f* and *g* are unknown, the $G^*$ can not be implemented. However, we can find the upper bound of *G* according to the system bounds.

$$\overline{G} = (\eta + F)\breve{g} + |1 - \breve{g}||\breve{u}| \tag{12}$$

This extreme $\overline{G}$ usually is too large and is hard to implement in physical applications. Moreover, it causes a heavily chattering phenomenon when the state trajectory crosses the sliding surface. Thus the value of *G* should be dynamically adjusted. In this study, the extension set is used to construct the value of *G* so that the extension controller can best estimate the optimal *G*.

# III. Extension Controller Design

Now, the extension controller is constructed as

$$u = \begin{cases} u_h, & S \in \mathrm{E_R} \\ u_{eq} + u_h, & S \notin \mathrm{E_R} \end{cases} \tag{13}$$

where $\mathrm{E_R}$ denotes the extension region. The required system specification is in the fitting region according to the basic idea of extension theory. For a dynamic control system, we should provide a control signal to force the *S* to move to the fitting region if the *S* falls in the extension region. Similarly, an appropriate control signal is needed once the *S* in the fitting region. It can be seen from (13), that the hitting control is activated when the *S* is in the extension region; however, equivalent control and hitting control are activated in the fitting region. As discussed above, we should properly design the *G* such that the resulted hitting control can preserve the system stability. Consequently, the parameter of *G* in the extension controller should be well adapted such that the state trajectory can be derived from extension region to the fitting region. Suppose that there exists a constant $\hat{G}_E$ and an optimal $G^*$ such that $e = \hat{G}_E - G^* > 0$ is minimized. Using the extension controller to achieve this task, the extension characteristic functions for each error state are shown in Fig. 1.

We assume that the gain parameter *G* results from:

$$G = \xi^T \mathbf{K} \tag{14}$$

where

$$\xi = [c_1, c_2, \cdots, c_n]^T \tag{15}$$

$$\mathbf{K} = [k_1, \ldots, k_n]^T \tag{16}$$

$$k_i = \frac{K(e^i)}{\sum_i K(e^i)} \tag{17}$$

Define

$$\hat{G}_E = \mathrm{x}^{*T}\mathbf{K} \tag{18}$$

$$\bar{\mathrm{x}} = \mathrm{x} - \mathrm{x}^* \tag{19}$$

A Lyapunov function is defined as

$$V = \frac{1}{2}\left(S^2 + \frac{1}{\gamma}\bar{\mathrm{x}}^T\bar{\mathrm{x}}\right) \tag{20}$$

Then

$$\begin{aligned}
\dot{V} &= S\dot{S} + \frac{1}{\gamma}\bar{\mathrm{x}}^T\dot{\bar{\mathrm{x}}} = S\left[f + (g\hat{g}^{-1} - 1)\breve{u} - g\hat{g}^{-1}G\,\mathrm{sgn}(S)\right] + \frac{1}{\gamma}\bar{\mathrm{x}}^T\dot{\bar{\mathrm{x}}} \\
&= S\left[f + (g\hat{g}^{-1} - 1)\breve{u}\right] - g\hat{g}^{-1}G|S| + \frac{1}{\gamma}\bar{\mathrm{x}}^T\dot{\bar{\mathrm{x}}} \\
&\qquad + g\hat{g}^{-1}G^*|S| - g\hat{g}^{-1}G^*|S| + g\hat{g}^{-1}\hat{G}_E|S| - g\hat{g}^{-1}\hat{G}_E|S| \\
&\le -\eta|S| - \breve{g}^{-1}\varepsilon|S| - \breve{g}^{-1}(\mathrm{x} - \mathrm{x}^*)^T\mathbf{K}|S| + \frac{1}{\gamma}\bar{\mathrm{x}}^T\dot{\bar{\mathrm{x}}} \\
&\le -(\eta + \breve{g}^{-1}\varepsilon)|S| + \frac{1}{\gamma}\bar{\mathrm{x}}^T(\dot{\bar{\mathrm{x}}} - \gamma\breve{g}^{-1}\mathbf{K}|S|)
\end{aligned} \tag{21}$$

The adaptation law can be chosen as

$$\dot{\bar{\mathrm{x}}} = \breve{g}^{-1}\gamma\mathbf{K}|S| \tag{22}$$

such that

$$\dot{V} \le -(\eta + \breve{g}^{-1}\varepsilon)|S| < 0 \tag{23}$$

This indicates that the extension control system is stable underlying adapting the parameter $\bar{\mathrm{x}}$. Note that the hitting control is modified as

$$u_h = -\hat{g}^{-1}G\,sat(S) \tag{24}$$

to smooth the control signal, where $sat(\cdot)$ is a saturation function.

Finally, the extension controller design procedure can be summarized as :

(a) Define the sliding surface S,

(b) Construct the equivalent control and hitting control according to (8), (9) and (14),

(c) Determine the extension region based on the $S$ to organize the extension controller,

(d) Adapt the gain parameter of hitting control according to (22),

(e) Complete the extension controller design.

# IV. Case Study

In this section, we apply the extension controller to control a simple nonlinear time-varying system. The desired reference input is $x_d = \sin(t)$.

$$\ddot{x} + (0.5 + 0.2\sin(t))\dot{x} + 3.2\cos x = (1 + 0.2\sin(x))u \tag{25}$$

Clearly, $\bar{g} = 1.2$ and $\underline{g} = 0.8$, so we have $\hat{g} = 0.98$ and $\breve{g} = 1.22$. The sliding line is defined as

$$S = \dot{e} + 2e \tag{26}$$

By (8), the equivalent control is

$$u_{eq} = \hat{g}^{-1}\hat{u} = \frac{1}{0.98}(-\sin(t) - 2\dot{e}) \tag{27}$$

The hitting control is

$$u_h = -\frac{1}{0.98} G \operatorname{sgn}(S) \tag{28}$$

where

$$G = \mathrm{x}^T \mathbf{K} \tag{29}$$

$$\dot{\mathrm{x}} = \hat{g}^{-1} g \mathbf{K} |S| = \frac{1}{1.22} g \mathbf{K} |S| \tag{30}$$

Finally, the extension region is defined as $E_R = [1, \infty)\ or\ [-1, -\infty)$. The simulation results for the trajectories of output, control signal and *S* are respectively shown in Fig. 2 – Fig. 4. It can be seen from the simulation results that the proposed extension controller can control the nonlinear time-varying system and provide good system performance in this simulation study.

# V. Conclusions

An extension controller using the design scheme of sliding mode control is proposed in this paper. Two regions in the extension controller, the regions of the fitting region and the extension, are developed to judge which control should be active. One hitting control is used when the *S* falls in the fitting region. The other hitting control is the equivalent control with the hitting control which is employed to derive the state trajectory toward the fitting region when the state is outside the fitting region. It is clear that the design process for extension controller using the sliding mode approach is easier than conventional approach. In addition, based on the conventional adaptive approach, the parameters for extension controller are dynamically adapted to guarantee the system stability. Simulation results show the effectiveness of the proposed control scheme of the extension controller.

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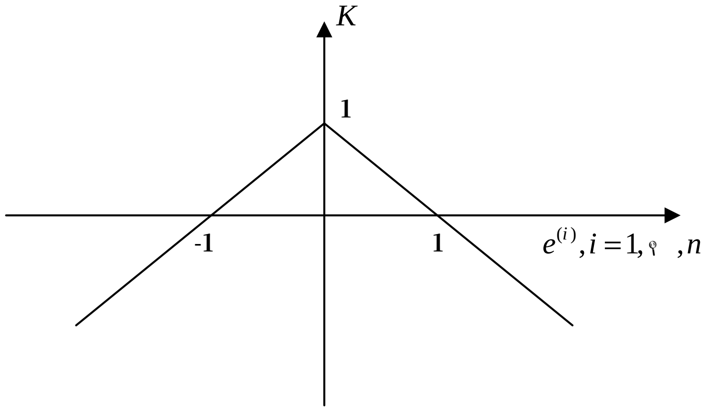


Fig 1 Extension characteristic function.

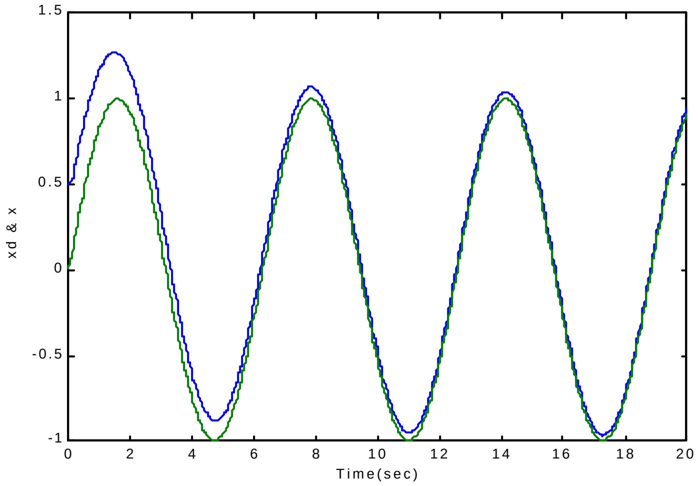


Fig 2 Output trajectory.

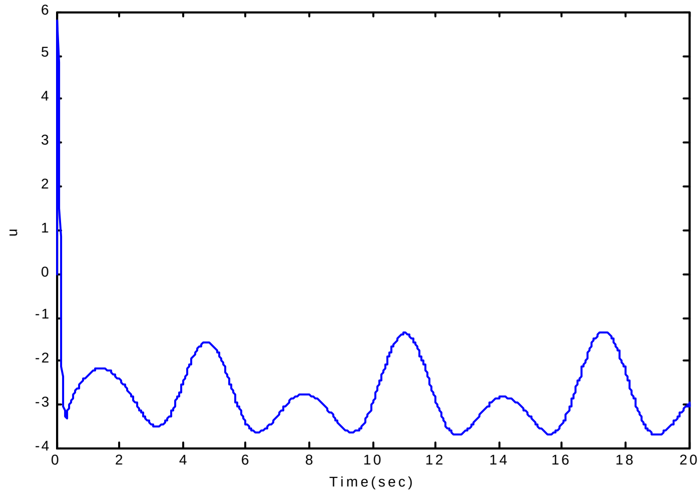


Fig 3 Control signal trajectory.

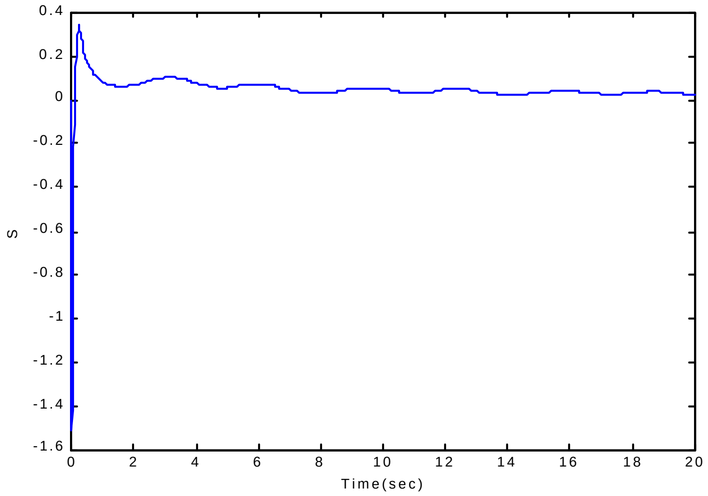


Fig 4 S trajectory.