基於模糊辨識之模糊控制器設計

吳德豐 [1] 蔡樸生 [2] 陳珍源 [3]

1. 宜蘭技術學院電機工程系講師
2. 中華技術學院電子工程系講師
3. 中華技術學院電子工程系教授

摘 要

本文首先利用蘇吉諾的模糊模型對一非線性系統中的輸入與輸出資料進行辨識，其根據 Stone-Weierstrass 定理證實，透過模糊推論的過程，模糊系統可精確地逼近任何連續函數。進而提出一套系統之辨識流程，包括以 Fuzzy C Mean 進行空間分割、以最小平方法作為初始參數之粗調、採用最陡梯度法進行參數微調、最後以一個性能指標函式作為模型評估之標準。最終目標在滿足模型最簡化以及精度上的兩大需求。接著，如何針對一個模糊模型的受控區間來設計一個模糊控制器是本文的第二個研究目的。藉由李雅普諾夫穩定條件與線性矩陣不等式所推導的穩定條件是目前研究的熱門課題，但如何有系統的尋找共同 P 解，以保證系統在李雅普諾夫下為漸近穩定仍是模糊控制器設計上之瓶頸。本研究將控制器之設計分為兩大步驟，首先將模糊模型中的每一條規則視為一個區域線性狀態方程式，針對每一個線性狀態模型設計相對應的狀態回授控制器。其次是建立一個全域穩定之條件，來取代共同 P 之求解，並保證所設計之模糊系統為全域穩定之系統。

關鍵詞 : 模糊建模、李雅普諾夫穩定條件、結構辨識、參數辨識、最陡梯度法，線性矩陣不等式

Fuzzy Control Design Based on Fuzzy Modeling

Ter-Feng Wu[1] Pu-Sheng Tsai[2] Jen-Yang Chen[2]

1. Lecturer, Department of Electrical Engineering, Ilan Institute of Technology
2. Lecturer, Department of Electronic Engineering, China Institute of Technology
3. Professor, Department of Electronic Engineering, China Institute of Technology

Abstract

In this paper, we first develop a procedure for constructing Takagi-Sugeno fuzzy systems from input-output pairs to identify nonlinear dynamic systems. The fuzzy system can approximate any nonlinear continuous function to any arbitrary accuracy that is substantiated by the Stone Weierstrass theorem. A learning-based algorithm is proposed in this paper for the identification of T-S models. Our modeling algorithm contains four blocks: fuzzy C-Mean partition block, LS coarse tuning, fine turning by gradient descent, and emulation block. The ultimate target is to design a fuzzy modeling to meet the requirements of both simplicity and accuracy for the input-output behavior. In the second part, we propose a discrete time fuzzy system that is composed of a dynamic fuzzy model and a fuzzy state feedback controller. This requires that for all the local linear models, a common positive-definite matrix P can be found to satisfy the Lyapunov stability criterion, although this is an extremely difficult problem for all systems. Thus in this paper, Fuzzy controller design is divided into two procedures. In the first step, we express the fuzzy model by a family of local state space models, and the controller is designed by state feedback control law for each local linear state space model. In the second step, we establish a global stability condition to guarantee the stability of the global closed loop system in order to circumvent the problem of determining the common P.

Key Words : Fuzzy Modeling, Lyapunov Stability Criterion, Structure Recognition, Parameter Identification, Maximum Gradient Descent, Linear Matrix Inequality

I. Introduction

The essential function of fuzzy systems is to formulate expert knowledge and experience in order to make a strategic decisions. Expert knowledge may be classified into two categories: conscious knowledge and subconscious knowledge. Conscious knowledge can be explicitly expressed in words, but subconscious knowledge is difficult to express precisely in words. When the expert is providing knowledge, we can view him as a black box and measure the input-output data pairs.

A fuzzy controller with expert knowledge or experience is sufficient to provide solutions to highly nonlinear, complicated, and unknown systems. This paper presents a systematic design method to identify a system model using a set of input-output data, thereby allowing the fuzzy model to satisfy the requirement of accuracy and minimum rules under the cluster analysis. In addition, this paper proposes a design for the fuzzy controller based on fuzzy models, thus creating the guidelines for the global stability of a fuzzy system under the Lyapunov stability criterion. According to the Stone-Weierstrass theorem, a fuzzy system is capable of approximating any continuous function [1] and can be used as the basis for fuzzy modeling theory. For the fuzzy identification, the modeling architecture presented by Takagi and Sugeno in 1985 [2] is becoming increasingly important and has been successfully applied to nonlinear modeling [3]. The output of a T-S model is a linear combination of input variables, and this model can be represented as state equations, which are more suitable for analysis of stability and robustness.

Fuzzy modeling can be divided into structure recognition and parameter identification. In structure recognition, the pattern recognition technique can be used to partition the state space of input variables. Then, the similarity and cluster methods from fuzzy theory can be used to effectively obtain the minimum number of rules and the distribution of membership functions, which are thus beneficial to the structure recognition [4]. Significant achievements have already been made with respect to the cluster analysis. Fuzzy C-Mean algorithm is the most commonly used cluster law [5]. Furthermore, R. Hathaway *et al.* presented Fuzzy C-Regression (FCRM) to deal with problems related to high dimension classification. Wong *et al.* substituted the gray correlation between data for the difference in data as the measurement standard for classification [6]. Wu *et al.* considered problems related to classification under the structure of linear regression. For parameter identification, they pick the optimal initial parameters by a genetic algorithm; and then adjust the parameters in the model by the maximum gradient descent [7]. Sungkwun *et al.* tuned parameters in the model by the least squares method and presented an evaluation function to modify the rules of tuning [8]. The current paper proposes a systematic fuzzy modeling procedure in the following steps: partition blocks of measurement data based on cluster analysis; then coarse tuning by the least squares method; then fine tuning by the maximum gradient descent; and finally assess the accuracy of the model by the evaluation function. If the accuracy is not sufficient, the number of rules are increased, which enables re-partitioning of space.

A fuzzy controller is a rule-based nonlinear controller, and in this paper a T-S model obtained from fuzzy modeling is the controlled plant. In contrast, E. Levin *et al.* presented an intuitional design method known as anti-T-S model controller, which intends to create an anti-model as the controller [9]. Another straightforward design method is the so-called T-S rules reflective controller, which is designed based on the 1 to 1 mapping of rules for the model being identified [10]. However, this method is not suitable if there are too many rules for the fuzzy model. To analyze the dynamic characteristics, Takagi and Sugeno presented stability criteria for both the controlled region of the T-S model and the closed-loop system under the T-S controller [11]. In this method, the fuzzy system is globally stable if each rule of this system is considered a sub-system for the purpose of finding a positive definition, a symmetric matrix P, such that all sub-systems must meet Lyapunov stability criterion. The discrete Lyapunov stability criteria were derived from the Linear Matrix Inequity (LMI) to find solutions [12]. However, once the number of rules increases, finding solutions for P becomes a difficult task.

After Takagi and Sugeno presented the stability criterion in 1996, searching for a common P solution to guarantee to the system stability became a popular topic. For example, Ma *et al.* considered the design of fuzzy controller and fuzzy observer simultaneously under the fuzzy model; and demonstrated that both can be designed independently in compliance with the Principle of Separation [13]. Joh proposed a recursive method to find the solution of P systematically in order to obtain the boundaries satisfying the stability criterion of all syb-systems, thereby facilitating the discussion of the robustness of system under the change of parameters [14]. In another study, Ying [15] transformed the consequence-parts of the rules of T-S model into a common basis; So as soon as a P satisfying a certain rule is found, all sub-systems will be satisfied. In Narendra *et al.* [16], the fuzzy system was converted into a LTV system, and the stability criterion for switching systems was presented. In a later study, Caoy *et al.* [17] presented another global stability guideline to replace the repeated solving of common P. Zak [18] designed one model rule only and considered other rules as uncertainty of the system, showing that the system is globally stable if the uncertainty satisfies a so-called compatibility criterion. Feng [19] attached a robust term to the fuzzy controller to compensate for influence on the system imposed by the uncertainty.

The current study infers the criterion of global stability for a fuzzy system from the discrete model based on the fuzzy identification, and tightly combines the fuzzy modeling and fuzzy controller design, thus demonstrating the feasibility of the system architecture, as demonstrated through simulation.

II. Fuzzy Modeling

1. Takagi-Sugeno fuzzy model

The T-S model can be expressed as follows:

$$\begin{aligned} &If\ x(k)\ is A_1^i\ and\ .....and\, x(k-n+1)\ is A_n^i\ ,Then \\ &x^i(k+1)=a_0^i+a_1^i x(k)+...+a_n^i x(k-n+1), \end{aligned} \tag{1}$$

where $i=1,...,m$ represents the number of fuzzy rules, $j=1,...,n$ represents the number of input variables, and A_j^i represents the membership function of the i[th] rule and j[th] input variable. One characteristic of this model is that the consequence can be expressed as the linear combination of the input variables of the premise. In addition, in this the consequence is expressed as the linear combination of input variables of the premise.

2. Fuzzy Modeling

To obtain the T-S model corresponding to various types of input-output data, the process of fuzzy modeling can be divided into structural recognition and parameter identification. The purpose of structural recognition is to determine the number of input variables, thereby partitioning input space; determining the number of rules; building up the initial distribution of membership functions of input variables and the consequential parameters; and determining the architecture for the approximate models. Parameter identification is used to eliminate the difference between models and physical systems, thereby obtaining a complete and accurate model via a fine tuning algorithm of parameter. This paper partitions the modeling process into 4 blocks, as shown in Fig. 1.

A. Partition Block:

The blocks of input data and output data are partitioned first. This study adopts the fuzzy C-Mean cluster method incorporating the concept of optimization to partition blocks. Let $X=\{x_1,x_2,.....,x_n\}$ be the data to be identified, $c\in\{2,3,.....,n-1\},1\le k\le n,1\le i\le c$ be the number of clusters, Matrix $U=[u_{ik}]$ be the level of membership of k[th] data to i[th] cluster and $V=(v_1,v_2,.....,v_c)$ be the central vector of each cluster.

According to the following operation:

(a) objective: to find $U=[u_{ik}]\in M_f$ and $V=(v_1,v_2,.....,v_c)$ such that

(b) minimize: $J(U,V)=\sum_{k=1}^{n}\sum_{i=1}^{c}(u_{ik})^m\|x_k-v_i\|^2,\ m\in(1,\infty)$ is a weighted value,

(c) subject to: $\sum_{i=1}^{c}u_{ik}=1,\quad \forall k=\{1,2\ldots,n\},\ u_{ij}\ge 0,\quad 1\le i\le n,\quad 1\le j\le c,$

we can locate

$$\begin{aligned} u_{ik}&=\frac{1}{\sum_{j=1}^{c}\left(\frac{\|x_k-v_i\|}{\|x_k-v_j\|}\right)^{\frac{2}{m-1}}},1\le i\le c,\ \ 1\le k\le n \\ v_i&=\frac{\sum_{k=1}^{n}(u_{ik})^m x_k}{\sum_{k-1}^{n}(u_{ik})^m},1\le i\le c \end{aligned} \tag{2}$$

For the requirement of minimum number of rules in structural recognition, an optimal balance between the minimum number of rules and approximate errors must be found. In the beginning, all data is divided into 2 categories (c=0) in order to construct the simplest model, and whether the system requirement can be satisfied is determined by the final model evaluation. If it fails, the number of partitioned blocks must be increased, which means increasing the number of rules for the fuzzy model.

3. Tuning parameters by the least squares method:

The model parameters are tuned by the least squares method in order to determine the initial parameters of the premise, as well the consequence of the fuzzy rules.

A. Parameters of membership function for premise:

As shown by Gauss distribution, parameters m_j^i and σ_j^i need to be determined.

$$A_j^i(m_j^i, \sigma_j^i) = \exp\left\{-(\frac{x_j - m_j^i}{\sigma_j^i})^2\right\}$$

$$m_j^i = \frac{\sum_{k=1}^{n} u_k^i x_{kj}}{\sum_{k=1}^{n} u_k^i} \quad , \quad \sigma_j^i = \sqrt{\frac{\sum_{k=1}^{n} u_k^i (x_{kj} - m_j^i)^2}{\sum_{k=1}^{n} u_k^i}} \tag{3}$$

where $i = 1, \cdots, c$ denotes the number of cluster (rules), and $u_k^i = u_{ik}$.

B. Setting up regression parameters for consequence

The output of the fuzzy model is the linear combination of input variables. The following recursive formulation of least square method determines the parameters for each rule *i*:

$$P^i = [a_0^i \quad a_1^i \quad \cdots \quad a_n^i]^T$$

$$y^i = a_0^i + a_1^i x_1 + a_2^i x_2 + ... + a_n^i x_n$$

$$y^i = X^T P^i \qquad i = 1, 2, \cdots, c$$

$$X = [1 \quad x_1 \quad \cdots \quad x_n]^T$$

$$P^i = [a_0^i \quad a_1^i \quad \cdots \quad a_n^i]^T$$

$$P_{k+1}^i = P_k^i + K_k [y_{k+1} - X_{k+1}^T P_k^i] \tag{4}$$

$$K_k = S_{k+1} X_{k+1} = \frac{S_k X_{k+1}}{(m_k^i)^{-1} + X_{k+1}^T S_k X_{k+1}} \tag{5}$$

$$S_{k+1} = [1 - K_k X_{k+1}^T] S_k \quad k = 1, 2, \cdots, n \tag{6}$$

4. Tuning parameters by the maximum gradient descent

In the maximum gradient descent, the difference between the fuzzy model and the object of identification is considered a performance index, pushing parameters upward for least errors, and thereby systematically adjusting the model parameters by recursive equation. Fundamentally, the learning effect of parameters or convergence speed is dependent on 3 factors – accuracy of structural recognition, quality of learning laws, and the parameter η_a of learning speed. The parameters to be tuned in this paper include the parameters (m_j^i, σ_j^i) of membership function in the rules of the premise, as well as the combinative parameters (a_j^i) of consequence. Initially, the error function is defined as $E = \frac{1}{2}[y_p(k) - y_m(k)]^2$, with $y_p(k)$ as the output sequence of the real system, and $y_m(k)$ as the output sequence of fuzzy model.

$$a_0^i(k+1) = a_0^i(k) - \eta_a \frac{\partial E(k)}{\partial a_0^i(k)} \tag{7}$$

$$a_j^i(k+1) = a_j^i(k) - \eta_a \frac{\partial E(k)}{\partial a_j^i(k)} \tag{8}$$

$$m_j^i(k+1) = m_j^i(k) - \eta_m \frac{\partial E(k)}{\partial m_j^i(k)} \tag{9}$$

$$\sigma_j^i(k+1) = \sigma_j^i(k) - \eta_s \frac{\partial E(k)}{\partial \sigma_j^i(k)} \tag{10}$$

Further, the chain rule is used to derive

$$\frac{\partial E(k)}{\partial a_0^i(k)},\ \frac{\partial E(k)}{\partial a_j^i(k)},\ \frac{\partial E(k)}{\partial m_j^i(k)},\ \frac{\partial E(k)}{\partial \sigma_j^i(k)}, \tag{11}$$

and the maximum gradient descent is used to find the learning laws as

$$a_0^i(k+1) = a_0^i(k) + \eta_a (y_p(k) - y_m(k)) \phi^i(k) \tag{12}$$

$$a_j^i(k+1) = a_j^i(k) + \eta_a (y_p(k) - y_m(k)) \phi^i(k) x_j(k) \tag{13}$$

$$m_j^i(k+1) = m_j^i(k) + \eta_m \frac{(y_p(k) - y_m(k))(y_m^i(k) - y_m(k)) \phi^i(k)(x_j(k) - m_j^i(k))}{(\sigma_j^i(k))^2} \tag{14}$$

$$\sigma_j^i(k+1) = \sigma_j^i(k) + \eta_s \frac{(y_p(k) - y_m(k))(y_m^i(k) - y_m(k)) \phi^i(k)(x_j(k) - m_j^i(k))^2}{(\sigma_j^i(k))^3} \tag{15}$$

III. Fuzzy State Feedback Controller

For convenience, the model in (1) is rewritten as follows:

$$\begin{aligned} &If\ x(k)\ is\ A_1^i\ and\ .....and\ x(k-n+1)\ is\ A_n^i\ \ Then \\ &x^i(k+1) = a_1^i x(k) + \cdots + a_n^i x(k-n+1) + b^i u(k) \\ &\qquad = A_i x(k) + B_i u(k) \end{aligned} \tag{16}$$

where $A_i = \begin{bmatrix} a_1^i & a_2^i & \cdots & a_{n-1}^i & a_n^i \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ & & \cdots & & \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ and $x(k) = [x(k)\ \ x(k-1) \cdots\ x(k-n+1)],\ i = 1, \cdots, m$.

The state feedback fuzzy controller u is designed as follows:

$$\begin{aligned} &If\ x(k)\ is\ A_1^i\ and\ .....and\ x(k-n+1)\ is\ A_n^i \\ &Then\ \ u(k) = -K_i x(k) \end{aligned} \tag{17}$$

After the process of fuzzy inference, the closed-loop system is integrated as

$$x(k+1) = \sum_{i=1}^{m} \sum_{j=1}^{m} \lambda_i(x) \lambda_j(x)(A_i - B_i K_j) x(k) \tag{18}$$

[Theorem 1] If the fuzzy system is asymptotically stable at the equilibrium point, there must exist a common positive symmetric matrix P to satisfy:

$$(A_i + B_i K_j)^T P (A_i + B_i K_j) - P = -Q_{ij} \quad i, j = 1, 2, \cdots, r \tag{19}$$

where Q_{ij} is a positive definite matrix. Each rule of the fuzzy system is considered a sub-system. The controller must satisfy not only the local stability of all sub-systems, but also the global stability of the overall system. Therefore, how to find a common P matrix to satisfy all rules and the fuzzy state matrix between all rules is the bottleneck for the design of fuzzy controller.

IV. Stability Analysis for the Fuzzy System

In this paper, we attempt to establish the criterion of global stability for fuzzy system from a different viewpoint. In equation (17), assuming that r[th] rule has the highest weighting and $r = \arg \max[\mu_j(x)],\ j = 1,2\cdots m$, then the output of controller can be denoted as :

$$
\begin{aligned}
u(k) &= -K_r x(k) \\
x(k+1) &= \sum_{i=1}^{m} \mu_i [A_i x(k) + B_i u(k)] \\
&= \sum_{i=1}^{m} \mu_i [A_i x(k) - B_i K_r] x(k) \\
&= (\mu_r A_{rr} + \sum_{\substack{i=1 \\ i \neq r}}^{m} \mu_i A_{ir}) x(k) \\
&= [A_{rr} + \sum_{\substack{i=1 \\ i \neq r}}^{m} \mu_i (A_{ir} - A_{rr})] x(k) = [A_{rr} + \Delta A_{ir}] x(k)
\end{aligned} \tag{20}
$$

where $A_{ir} = A_i - B_i K_r$, $A_{rr} = A_r - B_r K_r$, and $\Delta A_{ir} = \sum_{\substack{i=1 \\ i \neq r}}^{m} \mu_i (A_{ir} - A_{rr})$.

According to Theorem 1, there must be a positive symmetric matrix P_r for K_r to satisfy

$$
A_{rr}^T P_r A_{rr} - P_r + 2I = 0 \tag{21}
$$

Further, define a Lyapunov equation $V_r[x(k)] = x^T(k) P_r x(k)$, so that $\Delta V_r = V_r[x(k+1)] - V_r[x(k)] < 0$ is the criterion for asymptotical stability. We obtain

$$
\begin{aligned}
\Delta V_r &= V_r[x(k+1)] - V_r[x(k)] \\
&= x^T(k+1) P_r x(k+1) - x^T(k) P_r x(k) \\
&= [(A_{rr} + \Delta A_{ir}) x(k)]^T P_r [(A_{rr} + \Delta A_{ir}) x(k)] \\
&\quad - x^T(k) P_r x(k) \\
&= x^T(k) [(A_{rr} + \Delta A_{ir})^T P_r (A_{rr} + \Delta A_{ir}) - P_r] x(k) \\
&= x^T(k) [-2I + \Delta A_{ir}^T P_r A_{rr} + \\
&\qquad A_{rr}^T P_r \Delta A_{ir} + \Delta A_{ir}^T P_r \Delta A_{ir}] x(k) \\
&= 2x^T(k) [-I + \sum_{i=1, i \neq r}^{m} Q_{ri} + \sum_{i=1 i, \neq r}^{m} R_{rij}] x(k)
\end{aligned} \tag{22}
$$

where $\sum_{i=1, i \neq r}^{m} \mu_i Q_{ri} = \dfrac{\Delta A_{ir}^T P_r A_{rr} + A_{rr}^T P_r \Delta A_{ir}}{2}$ and $\sum_{i=1, i \neq r}^{m} \mu_i \mu_j R_{rij} = \dfrac{\Delta A_{ir}^T P_r \Delta A_{ir}}{2}$

[Theorem 2] If a closed-loop discrete fuzzy system satisfies $\sum_{i=1, i \neq r}^{m} \mu_i \lambda_{\max}(Q_{ri}) + \sum_{i=1, i \neq r}^{m} \sum_{j=1, j \neq r}^{m} \mu_i \mu_j \lambda_{\max}(R_{rij}) < 1$**,** then this fuzzy system has global asymptotical stability where $\lambda_{\max}(Q_{ri})$ and $\lambda_{\max}(R_{rij})$ are the maximum eigenvalues of matrices Q_{ri} and R_{rij} , respectively .

V. Simulation Results

[Example 1] The approximate model presented by Wang and Mendel [1992] is applied to a real case of auto-parking:

$$x(t+1) = x(t) + \cos[f(t) + q(t)] + \sin[q(t)]\sin[f(t)]$$
$$y(t+1) = y(t) + \sin[f(t) + q(t)] - \sin[q(t)]\cos[f(t)]$$
$$f(t+1) = f(t) - \sin^{-1}[\frac{2\sin(q(t))}{b}]$$

where $x \in [0,20]$, $f \in [-90^o, 270^o]$, $q \in [-40^o, 40^o]$, and b denotes car length.

In this figure, f denotes the angle between the car's movement and the x axis, and q denotes the angle between the wheels and the y axis. The angle created by the rotation of steering wheel is limited as $q \in [-40^o, 40^o]$. We desired to identify the trajectory x(t) through a fuzzy model. The procedures are as follows:

***Step1**:* Select $x(t)$, $f(t)$, and $q(t)$ as possible input variables.

***Step2**:* Normalize all variables.

$$x(t) = \frac{x(t)}{20},\ f(t) = \frac{f(t)}{2p},\ q(t) = \frac{q(t)}{4/9p}$$

***Step3**:* Partition data by Fuzzy C-Mean and set 800 points as the size of data.

***inputs**:* $q(t) = -(2p/9) + (k \cdot p/180)$, $x(t)$, and $f(t)$

***Output**:* x(t+1)

***Step4**:* Let the number of clusters be 5.

***Step5**:* Simulation results are as follows:

In this figure, assume that angle q starts from -40^o and increases 0.1^o each increment time until 40^o, so the steering wheel starts from -40^o and moves around a circle. As the steering wheel is turned to face front gradually, the radius increases gradually until $q = 0^o$. As soon as the steering wheel faces forward, the car moves straightforward. Then the car starts circling as the steering wheel turns to the other side. The fuzzy model derived from cluster analysis makes a perfect approximating effect possible.

[Example 2] an object of control under T-S model is denoted by two rules as follows:

T-S Model Plant :

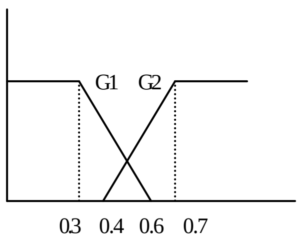
$$R^1: If\ x(k)\ is\ G_1, Then$$
$$x^1(k+1) = 2.18x(k) - 0.59x(k-1) - 0.603u(k)$$
$$R^2: If\ x(k)\ is\ G_2, Then$$
$$x^2(k+1) = 2.26x(k) - 0.36x(k-1) - 1.12u(k)$$

T-S Model Controller：

$$If\ x(k)\ is G_1, Then\ \ u^1(k+1) = k_1^1 x(k) + k_2^1 x(k-1)$$
$$If\ x(k)\ is G_2, Then\ \ u^2(k+1) = k_1^2 x(k) + k_2^2 x(k-1)$$

There, 4 parameters c_1^1, c_2^1, c_1^2, and c_2^2 were designed to stabilize the fuzzy closed-loop system.

Step1: Set up the membership function:



Step2 : Design the state feedback controller and urge all rules to satisfy the criterion of asymptotical stability; then select

$$K^1 = [k_1^1\ \ k_2^1] = [3.7813\ \ -0.1593]$$
$$K^2 = [k_1^2\ \ k_2^2] = [2.0833\ \ 0.0611]$$

Step3 : Find the positive and symmetric solutions for P_1 and P_2 in order to satisfy the Lyapunov equation under each rule

$$A_{rr}^T P_r A_{rr} - P_r + 2I = 0,\ \ r = 1,2$$

If the positive solutions of P_1, P_2 cannot be found, then redesign K^1, K^2 again in order to find a set of P as follows:

$$P_1 = \begin{bmatrix} 5.3148 & 0.176 \\ 0.176 & 3.2967 \end{bmatrix}$$

$$l_{\max}(Q_{12}) = 0.404, \quad l_{\max}(Q_{21}) = 1.221$$
$$l_{\max}(R_{122}) = 0.266, \quad l_{\max}(R_{211}) = 0.308$$

Step4 : Verify the global stability of the fuzzy system.

$$\sum_{i=1,j\neq r}^{m} m_i l_{\max}(Q_{ri}) + \sum_{i=1,i\neq r}^{m} \sum_{j=1,j\neq r}^{m} m_i m_j l_{\max}(R_{rij})$$

(1) $m_1 > m_2$:

$m_2 l_{\max}(Q_{12}) + m_2 m_2 l_{\max}(R_{122})$

$= 0.5*0.404 + 0.5*0.5*0.266 < 1$

(2) $m_2 > m_1$:

$m_1 l_{\max}(Q_{21}) + m_1 m_1 l_{\max}(R_{211})$

$= 0.5*1.221 + 0.5*0.5*0.308 < 1$

Step5 : Obtain simulation results (initial value x(k)=0.5, x(k-1)=0.5)

This figure demonstrates that this system is a stable fuzzy system.

VI. Conclusions

The fuzzy control architecture proposed in this paper is based on fuzzy modeling and has successfully incorporated the advantages of fuzzy dynamic model and fuzzy state feedback controller, and thus is beneficial to the tracking control of the reference model. The contributions made by this paper include: (1) Approximating an unknown system by constructing Takagi-Sugeno fuzzy systems model from input-output pairs, thereby building up the basic of theoretical analysis for fuzzy modeling; (2) Identifying the T-S model parameters by a learning-based algorithm contains four blocks: fuzzy C-Mean partition block, LS coarse tuning, fine turning by gradient descent, and emulation block.; (3) Meeting the requirements of both simplicity and accuracy for the input-output behavior by the proposed fuzzy design approach; (4) Implementing a discrete time full fuzzy system that is composed of a dynamic fuzzy model and a fuzzy state feedback controller, and finding a common positive-definite matrix P to satisfy the Lyapunov stability criterion; and (5) Avoiding the problem of determining the common P by establishing a global stability condition to guarantee the global stability of the closed loop system. Finally, simulation results for the trajectory tracking control of a mobile robot system show the effectiveness of the proposed control scheme of the TSK fuzzy controllers. In the future, researchs may incorporate the powerful learning ability of the neural network to adapt the parameters of various fuzzy basis functions, thereby eliminating the approximation errors.

References

1. L.X. Wang and J.M. Mendel, (Sep. 1992), “Fuzzy Basis Functions, Universal Approximation and Orthogonal Least-Squares Learning,” *IEEE Trans. On Neural Networks*, Vol.3, No.5, pp.807-814.
2. T. Takagi and M. Sugeno, (Jan./Feb. 1985), ”Fuzzy Identification of Systems and Its Application to Modeling and Control,” *IEEE Trans. On System, Man, and Cybernetics*, Vol. 15, No. 1, pp.116-132.
3. M. Sugeno and K. Tanaka, (1991), ”Successive Identification of a Fuzzy Model and Its Applications to Prediction of a Complex System,” *Fuzzy Sets and Systems*, Vol.42, pp.315-334.
4. M. Sugeno and G.T. Kang, 1988, “Structure Identification of Fuzzy Model,” *Fuzzy Sets and Systems*, Vol.28, pp. 15-33.
5. Antonio and Fernando Martin, (1999), “Membership functions in the fuzzy C-means algorithm” *Fuzzy Sets and System*, 101 (1999), pp. 49-58.
6. **翁慶昌、賴宏仁、陳嘉叢，「以灰色關聯度為基礎的聚類演算法」，淡江大學電機研究所。**

7. Baolin Wu and Xinghuo Yu, (2000), "Fuzzy Modelling and identification with genetic algorithm based learning" *Fuzzy Sets and System*, 113 (2000), pp. 351-365.
8. Sungkwun and Witold, (2000), "Identification of fuzzy systems by means of auto-tuning algorithm and its application to nonlinear systems" *Fuzzy Sets and System*, 115 (2000), pp. 205-230.
9. E. Levin, R. Gewirtzman and G.F. Inbar, (Feb. 1911), "Neural network architecture for adaptive system modeling and control," *IEEE Trans. on Neural Network*, Vol. 4, pp. 185-191.
10. T. Yamada and T.Yabuta, (Jan./Feb. 1993), "Dynamic System Identification Using Neural Networks," *IEEE Trans. on Syst.*, Man, and Cyber., Vol. 23, No. 1, pp. 36-47.
11. T. Takagi and M. Sugeno, (1992), "Stability Analysis and Design of Fuzzy Control Ssystems," *Fuzzy Sets Syst.*, Vol. 45, pp. 135-156.
12. Hua O.Wang and Kazuo Tanaka, (Feb. 1996), "An Approach to Fuzzy Control of Nonlinear Systems: Stability and Design Issues," *IEEE Trans. on Fuzzy Systems*, Vol. 1, No. 1, pp. 14-23.
13. Xiao-Jun Ma and Zeng-Qi Sun, (Feb. 1998), "Analysis and Design of Fuzzy Controller and Fuzzy Observer," *IEEE Trans. on Fuzzy Systems*, Vol. 6, No. 1, pp. 41-51.
14. J. Joh, Ye-Haw Chen, (Aug. 1998), "On the Stability Issues of Linear Takagi-Sugeno Fuzzy Models," *IEEE Tran. Fuzzy Syst.*, Vol. 6. No. 3.
15. Hao Ying, (May 1998), "Constructing Nonlinear Variable Gain Controllers via the Takagi-Sugeno Fuzzy Control," *IEEE Tran. Fuzzy Syst.*, Vol. 6. No. 2.
16. K.S. Narendra and J. Balakrishnan, (Dec. 1994), "A common Lyapunov function for stable LTI systems with commuting A-matrices," *IEEE Trans. Automat. contr.*, Vol. 39, pp. 2469-2471.
17. Shu-Guang Cao and Neville W.Rees, (Apr. 1999), "Analysis and Design of Fuzzy Control Systems Using Dynamic Fuzzy-State Space Models," *IEEE Tran. Fuzzy Syst.*, Vol. 7. No. 2. pp. 192-199.
18. Stanislaw H. Zak, (Apr. 1999), "Stabilizing Fuzzy System Models Using Linear Controllers," *IEEE Tran. Fuzzy Syst.*, Vol. 7. No. 2. pp236-240.
19. G. Feng, S.G. Cao, N.W.Rees, C.K. Chak, (1997), "Design of Fuzzy Control Systems with Guaranteed Stability," *Fuzzy Sets and System*, 85 (1997) 1-10.

91 年 09 月 16 日投稿
91 年 10 月 12 日接受

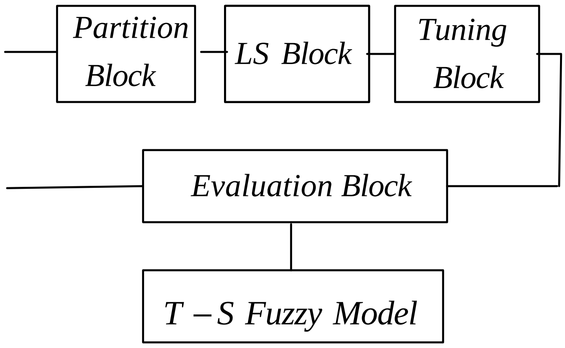


Fig 1 Block Diagram of Fuzzy Modeling.

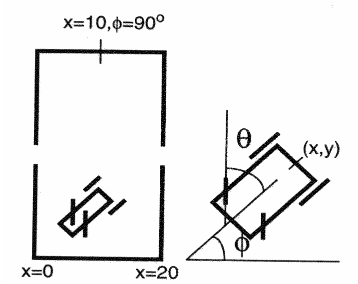


Fig 2 Mobile Robot Configuration.

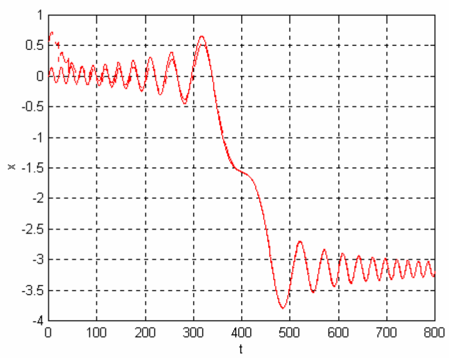


Fig 3 Fuzzy modeling of car's trajectory on x axis with '-' denoting system output and '--' denoting model output.

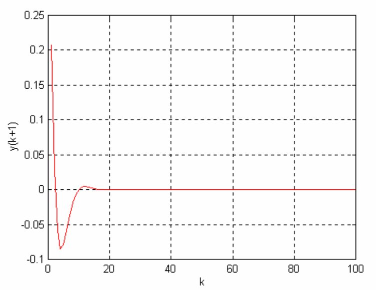


Fig 4 System response.