

簡易-T 型式模糊控制器之法則庫的簡化

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摘 要

在此文中，我們將提出一方式來簡化簡易-T 型式模糊控制器之法則庫。主要的作法是將簡易-T 型式模糊控制器輸入變數之模糊集合的歸屬函數加以修改，以定出所謂的多餘 (redundant) 模糊集合。將這多餘模糊集合去除，可產生具有較少數目之法則的等效模糊控制器。我們亦將證明此等效模糊控制器與原模糊控制器具有相同的輸入-輸出對應，來顯示此簡化方式之正確性。

關鍵詞：模糊控制器、多餘模糊集合。

Redundant Fuzzy Sets of The Easy-T Type Fuzzy Controllers

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Abstract

In this paper, an approach to indicate the redundant input fuzzy sets of the fuzzy rule base of the single input single output Easy-T type fuzzy controllers is proposed. The main idea is to modify the membership functions of fuzzy sets of input variables of the Easy-T type fuzzy controllers to point out the redundant fuzzy sets. The equivalent fuzzy controllers of the Easy-T type fuzzy controllers are constructed to have the redundant fuzzy sets been taken off. Thus, less number of fuzzy if-then rules are needed for the fuzzy rule base of the equivalent fuzzy controllers. The justification of this modification approach is provided to show that the equivalent fuzzy controllers and original Easy-T type fuzzy controllers have the same input-output mappings.

Keywords: Fuzzy controller, Redundant fuzzy sets.

I. Introduction

For the past several years, great efforts have been devoted to apply the fuzzy logic techniques [8] to the control systems [2], [3], [4], [6], [7]. And the fuzzy logic control have been successfully utilized in the complex ill-defined processes [5] with better performance than that of a PD controller in terms of shorter rise time and no overshoot. In many applications, the fuzzy controllers are constructed with the orthogonal triangular or trapezoidal membership functions, the algebraic product inference operator, and the centroid defuzzification technique. Since this type of fuzzy controllers has an easy defuzzification formula and triangular or trapezoidal membership functions, it is called Easy-T type fuzzy controller in our study. It is known that one of the important problems involved with the design of fuzzy logic controllers is the complexity of fuzzy controllers. The complexity of fuzzy controllers increases as the number of fuzzy if-then rules increases.

In this paper, an approach to point out the redundant fuzzy sets from the structures of the fuzzy rule base of the single input single output (*SISO*) Easy-T type fuzzy controllers is proposed. Moreover, the conditions for the input fuzzy sets of the multiple input single output (*MISO*) Easy-T type fuzzy controllers to be considered as redundant fuzzy sets are also derived. With the redundant fuzzy sets taken off, the equivalent fuzzy controllers of the Easy-T type fuzzy controllers are constructed. And there are less number of fuzzy if-then rules needed for the fuzzy rule base of the equivalent fuzzy controllers. In order to have some of the input fuzzy sets of the Easy-T type fuzzy controllers become redundant fuzzy sets, the membership functions of the corresponding input fuzzy sets are modified. The justification of this modification approach is provided to show that the input-output mappings of the equivalent fuzzy controllers and original easy T-type fuzzy controllers are the same.

II. The Easy-T Type Fuzzy Controller

In this section, the definition of the Easy-T type fuzzy controller (*ETFC*) are presented. Let the universe of discourse of a variable x be fuzzily partitioned into fuzzy sets F_i with membership functions $f_i, i=1,2,3,\dots,n$. If the condition

$$\sum_{i=1}^n f_i(x) = 1, \quad \forall x \quad (1)$$

is satisfied, the fuzzy partition of the variable x is said to have the property of orthogonality [1]. Then the fuzzy controller based on the fuzzy if-then rules

$$R_j: \text{If } x_1 \text{ is } A_{1j}, \dots, x_m \text{ is } A_{mj}, \text{ then } y \text{ is } B_j.$$

is defined to be an Easy-T type fuzzy controller if

1. the universe of discourse of every input variable $x_i, i=1,2,\dots,m$, is fuzzy partitioned with orthogonality.

2. the membership functions of fuzzy sets of the input variables of the fuzzy controller are either trapezoidal or triangular shapes with maximum membership value one.
3. the algebraic product is used as the fuzzy inference operator.
4. the crisp output is obtained with the centroid defuzzification technique.

where A_{ij} is one of fuzzy sets for the input variable x_i . With the definitions above, the modification approach for the indication of redundant fuzzy sets of Easy-T type fuzzy controller is detailed in the next two sections.

III. Redundant Fuzzy Sets of *SISO ETFC*

The redundant fuzzy sets for the Easy-T type fuzzy controller with single input and single output are depicted in this section. Since the membership functions for the fuzzy sets of input variables of the Easy-T type fuzzy controller are allowed to be either triangular or trapezoidal shapes, the discussion of this section is separated into two subsections as follows. For the rest of the paper, the abbreviations *ETTRI* and *ETTRA* are used to represent the Easy-T type fuzzy controllers with triangular and trapezoidal membership functions, respectively.

Redundant Fuzzy Sets of The *ETTRI*

Let the universe of discourse of the input variable x of *ETTRI* be fuzzily partitioned into n fuzzy sets F_j , $j=1,2,\dots,n$, with the membership functions $f_j(x)$'s shown in Figure 1. And the generality of the structure of the membership functions in Figure 1 is presented in the following proposition.

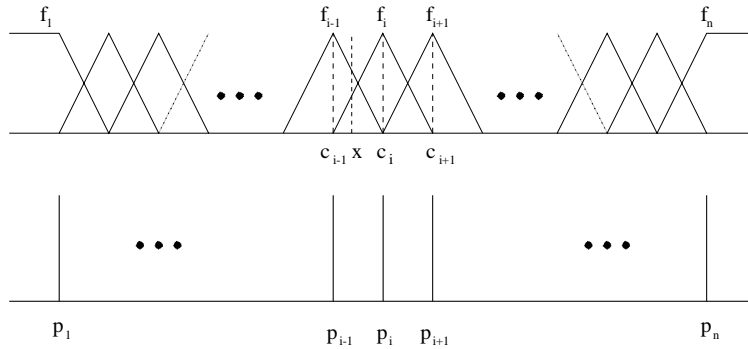


Figure 1. Membership functions of *ETTRI*.

Proposition 1. *Let the universe of discourse of the input variable x of a fuzzy controller be fuzzily partitioned into n fuzzy sets, F_j 's, with membership functions, $f_j(x)$'s ($j=1,2,\dots,n$), respectively. If the fuzzy partition has the property of orthogonality, and the membership functions, $f_j(x)$'s, are triangular with maximum membership value one, then for every point of x , the number n_f of $f_j(x)$'s such*

that $f_j(x) > 0$ is a positive integer and $n_f \leq 2$.

With the general structure of the membership functions of the input variable x , the membership functions (cf. Figure 1) of the output variable y , the algebraic product inference operator, and the centroid defuzzification method, the crisp output of *ETTRI* is obtained as

$$y = \frac{\sum_{l=1}^n p_l f_l(x)}{\sum_{l=1}^n f_l(x)} \quad (6)$$

where f_l denotes the input membership function, $p_l, l \in \{1, 2, \dots, n\}$ is the point at which the membership function of the corresponding output fuzzy set of the *ETTRI* achieves its maximum, and n denotes the number of fuzzy if-then rules. Note that the denominator of the output is always equal to one, and y is simplified to be

$$y = \sum_{l=1}^n p_l f_l(x) \quad (7)$$

Also, we can assume without loss of generality that the $p_l, l=2,3,\dots,n-1$ in Eq. 7 satisfy $p_{l+1} > p_l > p_{l-1}$. Thus, for the input variable x being in the region $[c_{i-1}, c_i]$ in Figure 1, the output y of *ETTRI* is

$$y = p_{i-1} + \frac{p_i - p_{i-1}}{c_i - c_{i-1}}(x - c_{i-1}) \quad (8)$$

Also, when x is in the region $[c_i, c_{i+1}]$, the output y of *ETTRI* is

$$y = p_i + \frac{p_{i+1} - p_i}{c_{i+1} - c_i}(x - c_i) \quad (9)$$

If the slopes of the two lines with equations as in Eq.8 and Eq.9 are equal,

$$\frac{c_{i+1} - c_i}{c_i - c_{i-1}} = \frac{p_{i+1} - p_i}{p_i - p_{i-1}} \quad (10)$$

then the membership function f_i can be replaced by the modifications of the membership functions f_{i-1} and f_{i+1} in Figure 2 and the fuzzy set F_i is considered to be a redundant fuzzy set.

To justify this, the output y of *ETTRI* for the input variable $x \in [c_{i-1}, c_i]$ with the modified membership functions in Figure 2 is derived as

$$y = p_{i-1} + \frac{p_{i+1} - p_{i-1}}{c_{i+1} - c_{i-1}}(x - c_{i-1}) \quad (11)$$

By using Eq.10, we have

$$\frac{p_i - p_{i-1}}{c_i - c_{i-1}} = \frac{p_{i+1} - p_{i-1}}{c_{i+1} - c_{i-1}} \quad (12)$$

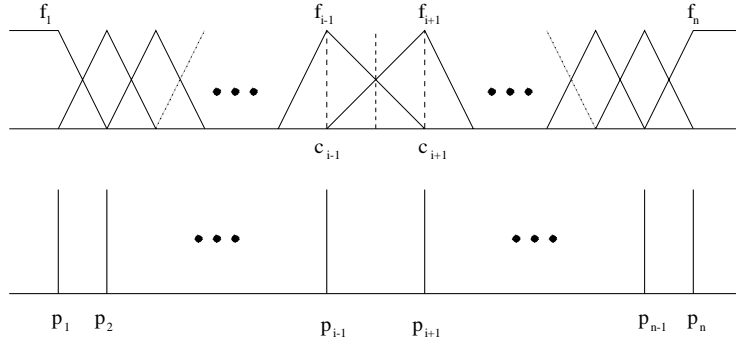


Figure 2. Modified membership functions, f_{i-1} and f_{i+1}

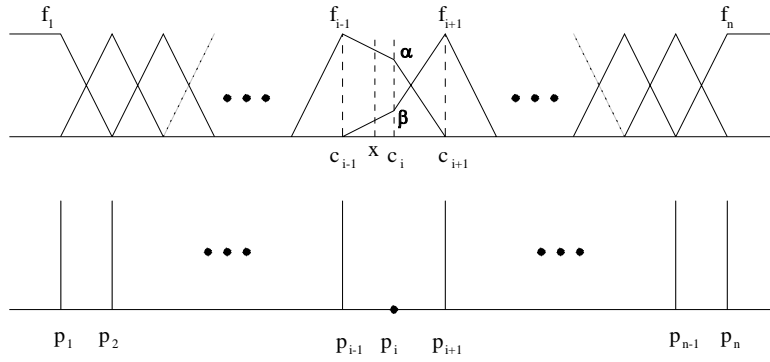


Figure 3. Modified f_{i-1} and f_{i+1} with α and β

From Eq.8, Eq.11, and Eq.12, the output of *ETTRI* for the input variable $x \in [c_{i-1} c_i]$ with the new structure of membership functions in Figure 2 is equal to the output of *ETTRI* for the input variable $x \in [c_{i-1} c_i]$ with the original structure of membership functions in Figure 1. The same result can be obtained for the input variable $x \in [c_i c_{i+1}]$. Therefore, the fuzzy set F_i is justified as a redundant fuzzy set. Furthermore, if the fuzzy set F_i of the input variable x of *ETTRI* is a redundant fuzzy set, then the fuzzy if-then rule

$$\text{If } x_1 \text{ is } F_i, \text{ then } y \text{ is } G_i.$$

is a redundant fuzzy if-then rule. If the fuzzy set G_i of the output variable y of *ETTRI* is only used in this redundant fuzzy if-then rule, then G_i is also a redundant fuzzy set. By taking off the fuzzy set F_i , the number of fuzzy if-then rules needed for the equivalent fuzzy controller is reduced. And the implementation of the original Easy-T type fuzzy controller can be simplified with the structure of the equivalent fuzzy controller.

In general, the condition in Eq.10 is not necessary for the F_i above to be considered as a redundant fuzzy set of the input variable x of *ETTRI*. With the membership functions f_{i-1} and f_{i+1} being modified as in Figure 3, the α and β are selected to satisfy the following relationship

$$\alpha + \beta = 1 \quad \text{and} \quad \frac{\alpha}{\beta} = \frac{p_{i+1} - p_i}{p_i - p_{i-1}} \quad (13)$$

Note that we can always find the proper α and β such that the Eq.13 is satisfied. Now the output y of

ETTRI with input variable $x \in [c_{i-1} c_i]$ can be derived by using the modified membership functions (f_{i-1} , f_{i+1}) and the corresponding output values (p_{i-1} , p_{i+1})

$$\begin{aligned} y &= p_{i+1} \frac{(x - c_{i-1})\beta}{c_i - c_{i-1}} + p_{i-1} \left(1 - \frac{(x - c_{i-1})(1 - \alpha)}{c_i - c_{i-1}}\right) \\ &= p_i \frac{x - c_{i-1}}{c_i - c_{i-1}} + p_{i-1} \left(1 - \frac{x - c_{i-1}}{c_i - c_{i-1}}\right) \end{aligned} \quad (14)$$

By utilizing the relationship of α and β , the output y of *ETTRI* for $x \in [c_i c_{i+1}]$ is obtained as

$$\begin{aligned} y &= p_{i+1} \left(\beta + \frac{(x - c_i)(1 - \beta)}{c_{i+1} - c_i}\right) + p_{i-1} \left(\alpha - \frac{(x - c_i)\alpha}{c_{i+1} - c_i}\right) \\ &= p_i + \frac{p_{i+1} - p_i}{c_{i+1} - c_{i-1}} (x - c_i) \end{aligned} \quad (15)$$

Since the output y in Eq.14 is equal to the y in Eq.8 and the output y in Eq.15 equals the y in Eq.9, we know that the effect of the membership function f_i of the fuzzy set F_i can be substituted with the modifications of adjacent membership functions f_{i-1} and f_{i+1} . Then the fuzzy set F_i is considered to be a redundant fuzzy set and can be taken off. Likewise, it is straightforward to find that more than one fuzzy sets of the input variable x of the *ETTRI* can be considered to be redundant fuzzy sets. To summarize the discussion of this subsection, the Proposition 2 is provided below.

Proposition 2 For a *SISO Easy-T type fuzzy controller ETTRI* with the membership functions presented in the Figure 1, the fuzzy set F_i can be considered as a redundant fuzzy set. That is, the fuzzy set F_i can be taken off with the membership function f_i replaced with the modified membership functions f_{i-1} and f_{i+1} for the fuzzy sets F_{i-1} and F_{i+1} (c.f. Figure 3), respectively. And the equivalent fuzzy controller without F_i will have less number of fuzzy if-then rules than the original fuzzy controller.

As one special case, c_i and p_i , $i=1,2,\dots,n$, are both equal spaced, that is,

$$c_i - c_{i-1} = \alpha_c, \quad p_i - p_{i-1} = \alpha_p, \quad i = 2,3,\dots,n \quad (16)$$

and α_p , α_c are constants. Then it is easy to know that the fuzzy controller *ETTRI* always can be simplified to have only two fuzzy sets. Thus, the computation complexity is greatly reduced.

Redundant Fuzzy Sets of The *ETTRA*

Let the universe of discourse of the input variable x of *ETTRI* be fuzzily partitioned into n fuzzy sets F_j , $j=1,2,\dots,n$, with the membership functions $f_j(x)$'s shown in Figure 4. The proposition 1 can also be applied to show the generality of the structure of the membership functions in Figure 4.

To indicate that the fuzzy set F_i with the membership function f_i in Figure 4 is a redundant fuzzy set, the Proposition 3 is presented. And the Proposition 3 can be shown with the same approach used to denote the redundant fuzzy sets for *ETTRI* in the subsection 3.1.

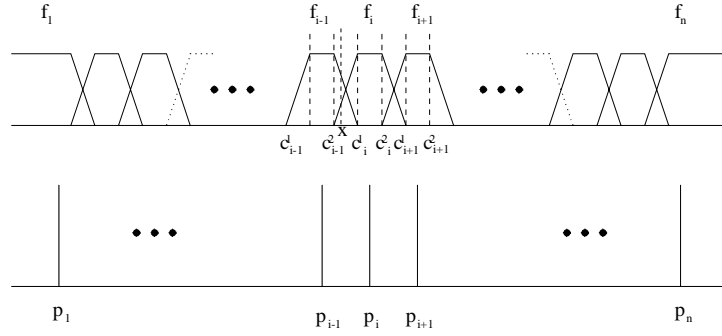


Figure 4. Membership functions of *ETTRA*

Proposition 3. *The fuzzy set F_i with the membership function f_i expressed in the Figure 4 can be considered as a redundant fuzzy set. Let $\alpha + \beta = 1$ and*

$$\frac{\alpha}{\beta} = \frac{p_{i+1} - p_i}{p_i - p_{i-1}} \quad (19)$$

the fuzzy set F_i can be taken off with the membership function f_i replaced with the modified membership functions f_{i-1} and f_{i+1} for the fuzzy sets F_{i-1} and F_{i+1} (c.f. Figure 5), respectively. And the complexity of the equivalent fuzzy controller without F_i is decreased.

Likewise, more than one fuzzy sets of the input variable x of the *ETTRA* can be considered as redundant fuzzy sets by using the same procedure. Moreover, in most of the cases, the number of the fuzzy sets needed for the output variable y of the equivalent fuzzy controller of *ETTRA* also decreases.

IV. Redundant Fuzzy Sets of *MISO ETFC*

In this section, the conditions for the fuzzy sets of the multiple inputs and single output Easy-T type fuzzy controller to be redundant fuzzy sets is provided. To make the discussions easier, the number of input variables of the Easy-T type fuzzy controller is taken to be two.

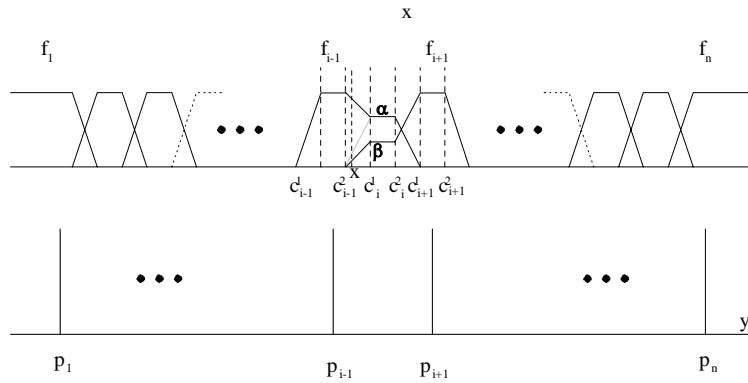


Figure 5. Modified f_{i-1} and f_{i+1} with α and β for *ETTRA*

Redundant Fuzzy Sets of The *MISO ETTRI*

To begin this subsection, we have the universes of discourses of the input variables x_1 and x_2 of *ETTRI* been fuzzily partitioned into n fuzzy sets F_{1i} and F_{2j} , $i, j = 1, 2, \dots, n$. And the structures of membership functions $f_{1i}(x_1)$'s and $f_{2j}(x_2)$'s are shown in Figure 6. Also, the the universe of discourse of the output variable y is fuzzily partitioned into $2n-1$ fuzzy sets P_k , $k=1, 2, \dots, 2n-1$, with the membership functions $p_k(y)$ presented in Figure 6. The fuzzy if-then rules for *ETTRI* are presented in the Table 1, and as usual, the diagonal cells of the Table 1 are designed to have the same output fuzzy sets. With the input and output membership functions as in Figure 6, the algebraic product inference operator \cdot , and the centroid defuzzification method, the crisp output y of *MISO ETTRI* is calculated as

$$y = \frac{\sum_{i=1}^n \sum_{j=1}^n p_{i+j-1} f_{1i}(x_1) f_{2j}(x_2)}{\sum_{i=1}^n \sum_{j=1}^n f_{1i}(x_1) f_{2j}(x_2)} = \sum_{i=1}^n \sum_{j=1}^n p_{i+j-1} f_{1i}(x_1) f_{2j}(x_2) \quad (18)$$

where f_{1i}, f_{2j} denote the membership functions, n denotes the number of fuzzy sets of input variable x_1 , and p_{i+j-1} is the point at which the membership function of corresponding output fuzzy sets of the *ETTRI* achieves its maximum. For most of the design of the *ETTRI*, we can assume without losing generality that the p_{i+j-1} , $i, j = 1, 2, 3, \dots, n$ in Eq. 18 satisfy $p_{\zeta} < p_{i+j-1}$; $\forall \zeta < i+j-1$, and $2n-1 \geq i+j-1 \geq 2$. The output y_i of *ETTRI* for the input variable $x = (x_1, x_2)$, in the region R_{ij} , $1 < i, j < n$, of the state space in Figure 6 is

$$y_{\gamma} = p_{i+j-2} \left(1 - \frac{x_2 - d_j}{d_{j+1} - d_j}\right) \left(1 - \frac{x_1 - c_{i-1}}{c_i - c_{i-1}}\right) + p_{i+j-1} \left(1 - \frac{x_2 - d_j}{d_{j+1} - d_j}\right) \frac{x_1 - c_{i-1}}{c_i - c_{i-1}} + p_{i+j-1} \left(1 - \frac{x_1 - c_{i-1}}{c_i - c_{i-1}}\right) \frac{x_2 - d_j}{d_{j+1} - d_j} + p_{i+j} \frac{x_1 - c_{i-1}}{c_i - c_{i-1}} \frac{x_2 - d_j}{d_{j+1} - d_j} \quad (19)$$

If the membership functions $f_{1(i-1)}$ and $f_{1(i+1)}$ are modified as in Figure 7 with the α and β satisfying $\alpha + \beta = 1$, then the output y_m of *ETTRI* with the same input variable x can be derived to be

$$y_m = p_{i+j-2} \left(1 - \frac{x_2 - d_j}{d_{j+1} - d_j}\right) \left(1 - \frac{(x_1 - c_{i-1})\beta}{c_i - c_{i-1}}\right) + p_{i+j-1} \left(1 - \frac{x_2 - d_j}{d_{j+1} - d_j}\right) \frac{(x_1 - c_{i-1})\beta}{c_i - c_{i-1}} + p_{i+j-1} \left(1 - \frac{(x_1 - c_{i-1})\beta}{c_i - c_{i-1}}\right) \frac{x_2 - d_j}{d_{j+1} - d_j} + p_{i+j} \frac{(x_1 - c_{i-1})\beta}{c_i - c_{i-1}} \frac{x_2 - d_j}{d_{j+1} - d_j} \quad (20)$$

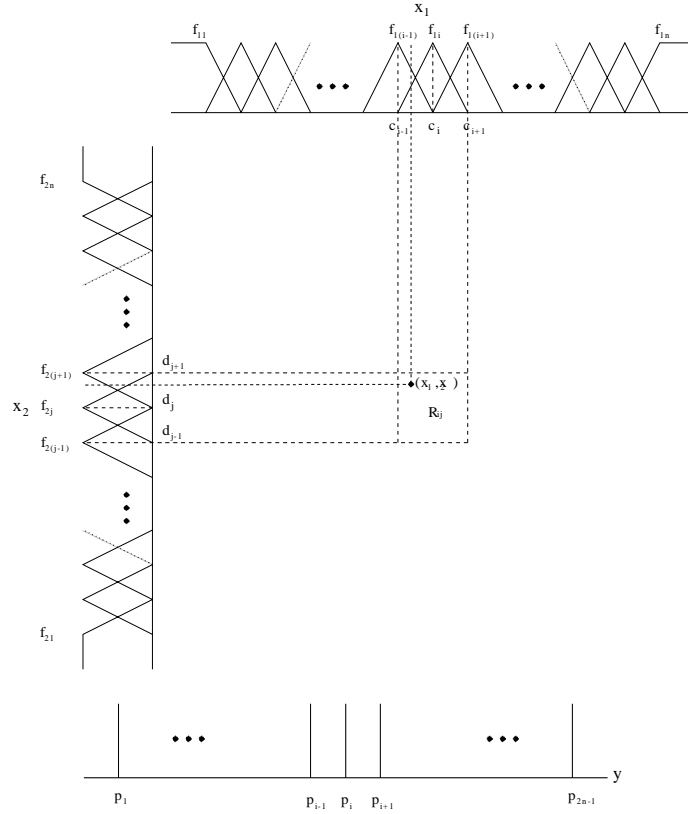


Figure 6. Membership functions for *MISO ETTRI*

Let y_d be the difference of y_r and y_m ,

$$\begin{aligned}
 y_d &= y_m - y_r \\
 &= \frac{x_2 - d_j}{d_{j+1} - d_j} \frac{x_1 - c_{i-1}}{c_i - c_{i-1}} (\beta(p_{i+j-2} - p_{i+j} + p_{i+j+1} - p_{i+j-1}) + p_{j+j-1} - p_{i+j-2} \\
 &\quad - p_{i+j} + p_{i+j-1}) + \frac{x_1 - c_{i-1}}{c_i - c_{i-1}} (\beta(p_{i+j} - p_{i+j-2}) - p_{i+j-1} + p_{i+j-2})
 \end{aligned} \tag{21}$$

It is found out that if

$$\frac{p_{i+j} - p_{i+j-1}}{p_{i+j+1} - p_{i+j-1}} = \frac{p_{i+j-1} - p_{i+j-2}}{p_{i+j} - p_{i+j-2}} \tag{22}$$

then we can have

$$\beta = \frac{p_{i+j} - p_{i+j-1}}{p_{i+j+1} - p_{i+j-1}} = \frac{p_{i+j-1} - p_{i+j-2}}{p_{i+j} - p_{i+j-2}} \tag{23}$$

such that $y_d = 0$. That is, with the modification of membership functions $f_{1(i-1)}$ and $f_{1(i+1)}$, the fuzzy set F_{1i} for the input variable x_1 of *ETTRI* is considered as a redundant fuzzy set in the region $c_i \geq x_1 \geq c_{i-1}$ and $d_{j+1} \geq x_2 \geq d_j$. When the input variable x is in the other subregion of R_{ij} , the conditions for the fuzzy set F_{1i} to be redundant fuzzy set can be found with the same approach. And the following proposition prepares the conditions for F_{1i} to be redundant fuzzy set with the input variable $x \in R_{ij}$.

$x_2 \backslash x_1$	F_{11}	..	$F_{1(i-1)}$	F_{1i}	$F_{1(i+1)}$..	F_{1n}
F_{2n}	P_n	P_{2n-1}
:	:						:
$F_{2(j+1)}$	P_{j+1}		P_{i+j-1}	P_{i+j}	P_{i+j+1}		:
F_{2j}	P_j		P_{i+j-2}	P_{i+j-1}	P_{i+j}		:
$F_{2(j-1)}$	P_{j-1}		P_{i+j-3}	P_{i+j-2}	P_{i+j-1}		:
:	:						:
F_{21}	P_1	..	P_{i-1}	P_i	P_{i+1}	..	P_n

Table 1. Fuzzy control rules

Proposition 4 For the MISO fuzzy controller with the membership functions as in the Figure 6 and the fuzzy if-then rules in the Table 1, if

$$\frac{P_{i+j} - P_{i+j-1}}{P_{i+j+1} - P_{i+j-1}} = \frac{P_{i+j-2} - P_{i+j-3}}{P_{i+j+1} - P_{i+j-3}}; \quad 1 \leq i + j - 3, i + j + 1 \leq 2n - 1 \quad (24)$$

then the fuzzy set F_{1i} is a redundant fuzzy set in the region R_{ij} with the membership functions $f_{1(i-1)}$ and $f_{1(i+1)}$ modified as in the figure 7 and

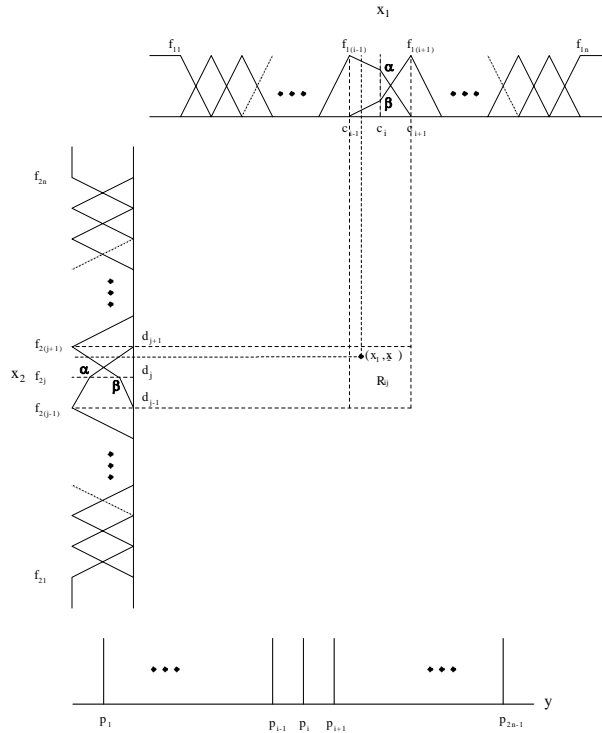


Figure 7. Modified $f_{1(i-1)}$, $f_{1(i+1)}$, $f_{2(j-1)}$, and $f_{2(j+1)}$.

$$\beta = \frac{P_{i+j-1} - P_{i+j-2}}{P_{i+j} - P_{i+j-2}} \quad (25)$$

Moreover, the Proposition 5,

Proposition 5 For the MISO fuzzy controller with the membership functions in the Figure 6 and the fuzzy if-then rules in the Table 1, if the same conditions as in the Eq. 24 are satisfied, then the fuzzy set F_{2j} is a redundant fuzzy set in the region R_{ij} with the membership functions $f_{2(j-1)}$ and $f_{2(j+1)}$ modified as in the Figure 7 and

$$\beta = \frac{P_{i+j-1} - P_{i+j-2}}{P_{i+j} - P_{i+j-2}} \quad (26)$$

can also be easily proved. However, unlike the SISO redundant fuzzy set, the fuzzy set F_{1i} is used in many other regions besides R_{ij} . And F_{1i} can not be taken off even it is a redundant fuzzy set in the region R_{ij} . In order to have F_{1i} be able to be taken off, F_{1i} needs to be a redundant fuzzy set in all the regions R_{ij} for $j=1, 2, \dots, n$. That is, if

$$\beta = \frac{P_{i+j-1} - P_{i+j-2}}{P_{i+j} - P_{i+j-2}}; \quad i > 1 \quad (27)$$

and $\beta_j = \beta_2 = \dots = \beta_n$, then F_{1i} is a redundant fuzzy set in all the regions $R_{ij}, j=1, 2, \dots, n$, and F_{1i} can be taken off. For example, if p_c 's, $2n-1 \geq \zeta \geq 1$, are equal spaced, then $\beta_j = \beta_2 = \dots = \beta_n = 0.5$, and β can be easily determined to be 0.5. Thus, with the modifications of adjacent membership functions $f_{1(i-1)}$ and $f_{1(i+1)}$, the fuzzy set F_i is considered to be a redundant fuzzy set. And the redundant fuzzy set F_i can be taken off. Moreover, if $c_i, d_i, i=1, 2, \dots, n$, and $p_i, i=1, 2, \dots, 2n-1$, are all equal spaced for the ETTRI with the fuzzy if-then rules in Table 1, i.e.,

$$\begin{aligned} c_i - c_{i-1} &= \gamma_c, \quad i = 2, 3, \dots, n, \\ d_i - d_{i-1} &= \gamma_d, \quad i = 2, 3, \dots, n, \\ p_i - p_{i-1} &= \gamma_p, \quad i = 2, 3, \dots, n, \end{aligned} \quad (28)$$

and $\gamma_c, \gamma_d, \gamma_p$ are constants. Then more than one redundant fuzzy sets exists for both input variables x_1 and x_2 . With the redundant fuzzy sets taken off step by step, the fuzzy controller ETTRI can be simplified to be an equivalent fuzzy controller with only two fuzzy sets. Thus, the complexity is greatly reduced.

Redundant Fuzzy Sets of The MISO ETTRA

Let the universes of discourses of the input variables x_1, x_2 of MISO ETTRA be fuzzily partitioned into fuzzy sets $F_{1i}, F_{2j}, i, j = 1, 2, \dots, n$, with the membership functions $f_{1i}(x_1), f_{2j}(x_2)$'s shown in Figure 8.

And the point $x=(x_1,x_2)$ is in the region R_{ij} . With the same approach used to develop the conditions for the input fuzzy sets of *MISO ETTRI* in subsection 4.1 to be redundant, the conditions for the input fuzzy sets of the *MISO ETTRA* to be redundant are presented in this subsection. For simplicity, these conditions are declared in propositions without derivation.

Proposition 6 For the *MISO* fuzzy controller *ETTRA* with the membership functions as in the Figure 8 and the fuzzy if-then rules in the Table 1, if

$$\frac{P_{i+j} - P_{i+j-1}}{P_{i+j+1} - P_{i+j-1}} = \frac{P_{i+j-2} - P_{i+j-3}}{P_{i+j+1} - P_{i+j-3}}; \quad 1 \leq i + j - 3, i + j + 1 \leq 2n - 1 \quad (29)$$

then the fuzzy set F_{1i} is a redundant fuzzy set in the region R_{ij} with the membership functions $f_{1(i-1)}$ and $f_{1(i+1)}$ modified as in the Figure 9 and

$$\beta = \frac{P_{i+j-1} - P_{i+j-2}}{P_{i+j} - P_{i+j-2}} \quad (30)$$

Proposition 7 For the *MISO* fuzzy controller with the membership functions in the Figure 8 and the fuzzy rules in the Table 1, if the conditions as in the Eq. 29 are satisfied, then the F_{2j} is a redundant fuzzy set in the region R_{ij} with the membership functions $f_{2(j-1)}$ and $f_{2(j+1)}$ modified as in the figure 9 and

$$\beta = \frac{P_{i+j-1} - P_{i+j-2}}{P_{i+j} - P_{i+j-2}} \quad (31)$$

Proposition 8 For the *MISO* fuzzy controller with the membership functions as in the Figure 8 and the fuzzy if-then rules in the Table 1, if

$$\beta = \frac{P_{i+j-1} - P_{i+j-2}}{P_{i+j} - P_{i+j-2}} \quad (32)$$

and $\beta_1=\beta_2=\dots=\beta_n$, then F_{1i} is a redundant fuzzy set in all the regions R_{ij} , $j=1,2,\dots,n$, and F_{1i} can be taken off.

V. Conclusions

In this paper, the fuzzy controllers constructed with the orthogonal triangular or trapezoidal membership functions, the algebraic product inference operator, and the centroid defuzzification technique are defined as the Easy-T type fuzzy controllers. With the membership functions of the corresponding input fuzzy sets modified, the redundant input fuzzy sets are indicated for the *SISO* Easy-T type fuzzy controllers. Also, the conditions for the input fuzzy sets of the *MISO* Easy-T type fuzzy controller to be redundant fuzzy sets are generated. In many application cases, with the redundant fuzzy sets taken off, the complexity of the fuzzy controllers is greatly reduced. And the

efficient equivalent fuzzy controllers are obtained.

VI. References

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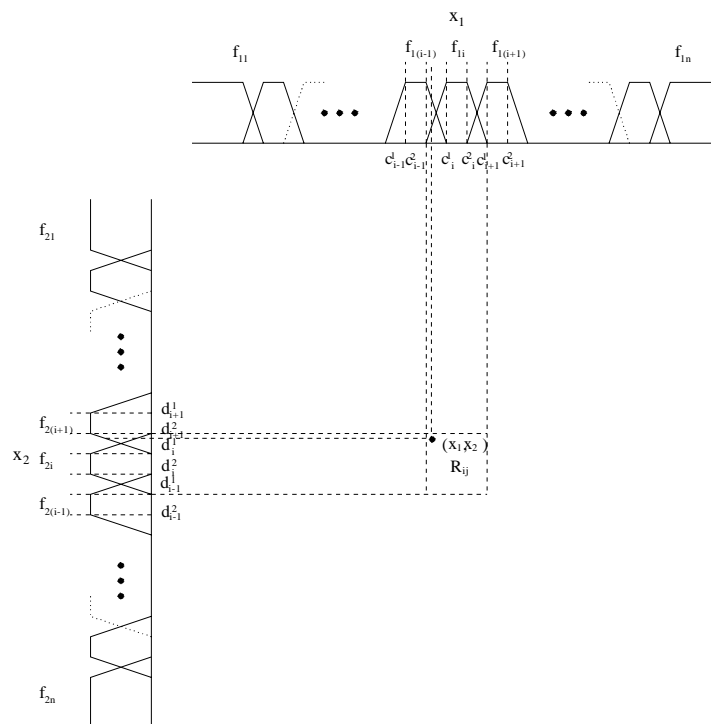


Figure 8. Input membership functions of *MISO ETTRA*.

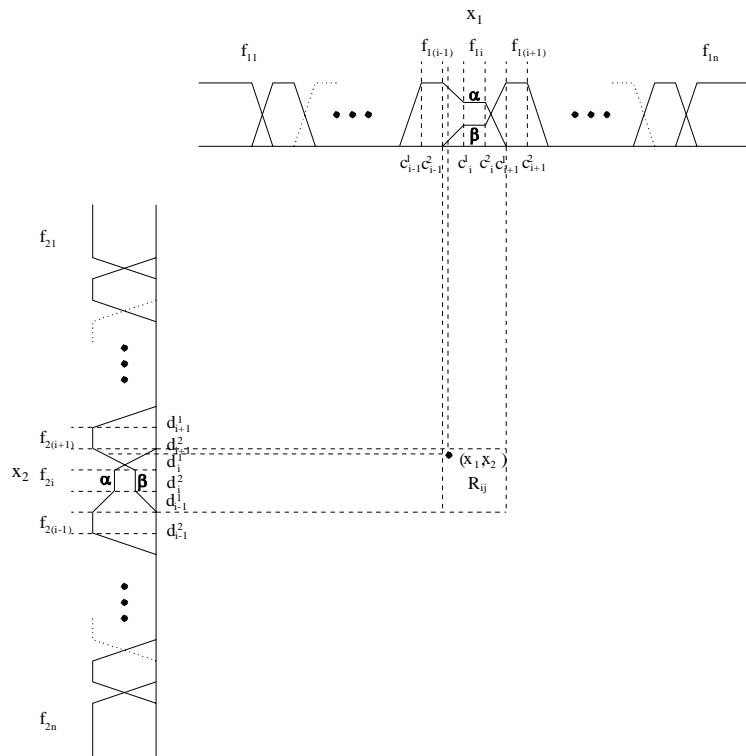


Figure 9. Modified $f_{1(i-1)}$, $f_{1(i+1)}$, $f_{2(i-1)}$, and $f_{2(i+1)}$.