藉由直覺式降階模型合成多目的控制器

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摘要

考量實務觀點，本文重新討論同時具有穩定及不穩定極點之常見系統的模型簡化控制問題。我們的工作不僅止於找出近似模型，更重要的是提出一個新方法以便藉由簡化模型進行全系統之分析與設計，並且免除從前對原受控器所需的所有條件限制。經由處理不穩定之子系統，本文提出於頻率域得到直覺式模型簡化之程序，接著推導出進化的全參數控制器設計法則以保證簡化系統及原始系統穩定，同時提供更多的自由度以滿足多目的性能要求。我們證明經由此設計，降階系統及原系統之閉迴路轉換函數將具有相似之外觀和響應。最後，我們也舉出一個廣為使用的實例以有效證明新技術於設計及分析控制任務上之優點。

關鍵字 :模型簡化，全參數控制器。

Synthesis of Multipurpose Controller via Intuitively Reduced-Order Models

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Abstract

From a practical viewpoint, the model-reduction control problem is revisited for a general class of systems. Further rather than just finding out the approximate model, the point of this work is to propose a new methodology for analysis and design of a system via its reduced-order model and without any previous restrictions on the plant. By dealing with the unstable subsystem, this paper presents procedures for intuitive model reduction in the frequency domain. An extended all-parametric controller synthesis algorithm is thus derived for stabilizing both the reduced and original plants as well as for providing more freedom to cope with multiple performance requirements. It is shown that the closed-loop transform matrices of these two systems have a similar outlook and their responses are also similar. Finally, the efficiency of this new methodology in facilitating the analysis and synthesis of control objectives is well illustrated by a widely used example.

Key Words: Model reduction, all-parametric controller.

I. Introduction

The model reduction problem has attracted attention in the past two decades due to its advantages in facilitating the analysis and design for a complex plant. A considerable amount of literature has been devoted to develop various methodologies. Among the recent techniques, balance truncation has become an indispensable tool [1,2]. In [3], a characterization of the solution for the H_∞ model reduction problem is reported. For brief overviews over more recent activities we refer the reader to [4]. However, almost all the derivations of these previous works are based on numerous assumptions on the target plant. It essentially needs to be stable and minimal phase. Moreover, although a great deal of work has been done on obtaining reduced-order models for complex systems, further investigation is need on the applicability of these models for designing controllers that would work well with the original systems.

As known, there are many situations where the system is inherently unstable and of relatively high dynamic complexity; i.e., a practical plant (such as chemical reactor or inductive motor) usually consists of not only stable but also unstable poles. Therefore, it is hard to apply most of the previous results in the real world. A few attempts have been made in recent to widen the class of linear systems that can be approximated [6-13]. However, the basic objective of control engineering is more than to just examine whether the behavior of systems is suitable or not. It is utterly imperative for industrial application to compensate the whole system to achieve an acceptable characteristic while it originally doesn' t. Since the reduced-order model embeds some errors in a certain frequency band, such a difficult task can or can' t be accomplished by a reducing model (especially for an unstable one) is still wrapped in mystery [14]. The model-reduction control problem is hence revisited here in frequency domain. For a general class of plant P containing both of LHP and RHP poles, there are a series of traditional questions need to be answered:

i) How to find a model G that has lower order but contains enough messages for P to facilitate the design of the controller?

ii) How to construct the controller C for G to stabilize the closed-loop subsystem as well as to achieve some desired performances? Such as reference input tracking, disturbance rejection, or arbitrary pole zero assignments?

iii) Does C still work well for the original plant P? i.e., Can C stabilize P? Can the real system track a desired signal and reject some undesired disturbances? And etc.

iv) How can we analysis the characteristic of the whole real system from this reducing-order model?

The keynote of our work is to analyze and control an unstable plant via its reducing-order model. It will be shown in the following sections that all these questions can be solved by a quite simple and straightforward approach. Section II gives some notations and preliminaries. By taking out the stable parts, section III presents procedures for intuitive model reduction. It is noted that our approach is directly derived in frequency domain instead of solving Lyapunov matrix equations, and it makes the proposed algorithm simpler and numerically more efficient. In section IV, the all-parametric controller synthesis algorithm for MIMO cases [5] is modified to become an extended version for compensating both of the reduced and original plants. Finally, an illustrative example is given in section V to verify the availability of this new methodology.

II. Preliminaries

Throughout this paper, we use the following notations. Italic A denotes a rational matrix or a linear operator. $\Re$ denotes the set of rational matrices whose elements are all proper stable or the norm space $\{A \mid \|A\| < \infty\}$, where $\|\bullet\|$ can be any Euclidean norm. As the plant is originally unstable, we consider an unit-feedback control scheme as Fig.1. Where $r, d, u,$ and y denote the exogenous input, disturbance, control signal, and plant output vectors, respectively. There are no restrictions on the plant. However, just for convenience, the plant P is $n \times m$ and the controller C is $m \times n$. Hence, PC is $n \times n$, and it is assumed of course that all other matrices (vectors) are of compatible dimensions.

The standard definition of internal stability is now given.

Definition 1 [5]

The feedback system of Fig.1. is denoted as S(P,C) and is internally stable or asymptotically stable if and only if

$$H(P,C) \equiv \begin{bmatrix} S & -SP \\ CS & I-CSP \end{bmatrix} \in \Re \tag{1}$$

where $S \equiv [I+PC]^{-1}$ is the sensitivity matrix.

In order to stabilize an open-loop unstable plant, Youla had proposed the most famous solution, all-parametric optimal controller. It plays an important roles in our analysis and the design for model-reduction control.

Lemma 1 (see Theorem 4.1.60 and Corollary 4.1.67 of [5])

Let the given plant P has a double coprime factorization

$$P = A^{-1}B = B_1 A_1^{-1} \tag{2}$$

where $A, B, A_1, B_1 \in \Re$ are left and right coprime matrices of P, respectively. For this double coprime factorization there exist transfer matrices $X, Y, X_1, Y_1 \in \Re$ satisfying the Bezout identity

$$\begin{bmatrix} X_1 & Y_1 \\ -B & A \end{bmatrix} \begin{bmatrix} A_1 & -Y \\ B_1 & X \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{3}$$

Then the stabilizing controller associated with a particular choice of admissible $K \in \Re$ possesses the transfer matrix

$$C = (Y + A_1 K)(X - B_1 K)^{-1} \tag{4}$$

and $\det(X - B_1 K) \neq 0$

III. Intutive Model Reduction

In this section, attention is restricted exclusively to a class of MIMO plants which consist of both stable and unstable modes. Let the system is described by the transfer matrices

$$P(s) = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22} & \cdots & P_{2m} \\ \cdots & & & \cdots \\ P_{n1} & \cdots & & P_{nm} \end{bmatrix}$$

where

$$P_{ij}(s) = \frac{h_{ij} \prod_{k=1}^{r_{ij}} (s - g_{ijk})}{\prod_{k=1}^{p_{ij}} (s - a_{ijk}) \prod_{k=1}^{q_{ij}} (s - b_{ijk})} \qquad \text{for i=1..n, j=1..m,} \tag{5}$$

and h_{ij} is the real gain and $a_{ijk}, b_{ijk}, g_{ijk}$ are the complex poles and zeros. Furthermore, we assume Re(a_{ijk}) ≥ 0 denotes the k' th unstable poles of P_{ij}(s). The method for model oreder reduction is determined by the criterion used for deciding what constitute the main feature of the systems. In general, these criterion may be represented in terms of a suitable norm and the optimal approximate model is yielded from

$$\min_G \|G - P\|_p \tag{6}$$

for $1 \leq p \leq \infty$ be some appropriate Euclidean norm. Unfortunately, such a widely-used methodology is obviously failed for unstable plants. We are thus forced to try an alternative approach which is based on obtaining a model of lower order such that its impulse response matches that of the original system in an acceptable manner. As pointed in [6], the low-frequency behavior is much more important in control system design and hence cannot be neglect. The reduction problem can be reformulated in terms of minimization of the error function

$$J=\int_0^\infty \left\| p(t)-g(t)\right\| dt \tag{7}$$

where $p(t)$, $g(t)$ are impulses response of plant P and model G , respectively. However, as P is unstable, the first thing should take into mind is to avoid J being infinity. From eqn. (5), its impulse response can be partitioned as

$$p_{ij}(t)=\quad \mathrm{L}^{-1}\left[\frac{b_{ij}(s)}{\prod_{k=1}^{p_{ij}}(s-\boldsymbol{a}_{ijk})}\right]+$$

$$\mathrm{L}^{-1}\left[\frac{\left(\boldsymbol{h}\prod_{k=1}^{r_{ij}}(s-\boldsymbol{g}_{ijk})-b_{ij}(s)\prod_{k=1}^{q_{ij}}(s-\boldsymbol{b}_{ijk})\right)\prod_{k=1}^{p_{ij}}(s-\boldsymbol{a}_{ijk})}{\prod_{k=1}^{q_{ij}}(s-\boldsymbol{b}_{ijk})}\right] \tag{8}$$

Where the second parts of $p_{ij}(t)$ is stable such that

$$\mathrm{L}^{-1}\left[\frac{\left(\boldsymbol{h}\prod_{k=1}^{r_{ij}}(s-\boldsymbol{g}_{ijk})-b_{ij}(s)\prod_{k=1}^{q_{ij}}(s-\boldsymbol{b}_{ijk})\right)\prod_{k=1}^{p_{ij}}(s-\boldsymbol{a}_{ijk})}{\prod_{k=1}^{q_{ij}}(s-\boldsymbol{b}_{ijk})}\right]\le c_{ij}\exp(-\boldsymbol{l}t_{ij}) \tag{9}$$

for all $t\ge 0$ and for some $c_{ij}>0,\ \boldsymbol{l}_{ij}>0$. This gives the idea that every plant with the same unstable poles should present a simmilar unstable response at time $t\to\infty$. In particular, several authors [8,15] have shown that: it is necessary to guarantee that the number of RHP poles of the reduced model is equal to that of the original unstable system. In order to retain this "main" unstable feature of the original system, eqn. (8) suggests that

$$a_{ij}(s)=\prod_{k=1}^{p_{ij}}(s-\boldsymbol{a}_{ijk}) \tag{10}$$

is a denominator candidate for approximating $P_{ij}(s)$. However, determination of the corresponding optimal numerator is still a difficult job. A traditional way (for example see [16]) is to balance and trucate the stable parts. Another possible solution [17] suggested that the parameters of the reducing model are determined by minimizing a weighted mean square error funtion involving the frequency response in some period. Nevertheless, in order to keep the algorithm simple and numerically more efficient, we suggest that

$$b_{ij}(s)=\boldsymbol{m}_{ij}\prod_{k=1}^{w_{ij}}(s-\boldsymbol{g}_{ijk}) \tag{11}$$

where

$$G_{ij}(s)=\frac{b_{ij}(s)}{a_{ij}(s)}$$

$$w_{ij} = \begin{cases} r_{ij} - q_{ij}, & for \quad r_{ij} > q_{ij} \\ p_{ij}, & for \quad r_{ij} \le q_{ij}, and \quad r_{ij} \ge p_{ij} \\ r_{ij}, & for \quad r_{ij} \le q_{ij}, and \quad r_{ij} < p_{ij} \end{cases} \tag{12}$$

$$m_{ij} = \lim_{s\to 0} \frac{P_{ij}(S)}{G_{ij}(S)} \tag{13}$$

and $\mathrm{Re}[g_i] \le \mathrm{Re}[g_j]$, for all j>i.

So that both G and $(P-G)$ are proper and one can ensure the similar steady-state value for the plant and model as well as can keep some coupling relationships of pole-zero.

Remark 1: Indeed, this intuitive approach of splitting the full order model into a stable and completely unstable subsystems is not new. In [11,12], they apply some existing reduction techniques to reduce the stable part and then yield a new subsystem by combining the unstable part. However, we claim that the main idea behind the model-reduction control technique is to produce a lower-order model with enough information so that it could replace the given system to facilitate the “design and analysis” of the controller. We will show that this objective can be well done by only taking the unstable subsystem.

IV. Main Results

Again, suppose we represent the intuitive model reduction process as

$$P = G + P_- \tag{14}$$

where G contain all the unstable poles of P, and $P_- \in \Re$. The model-reduction control technique is shown in the following Lemma.

Lemma 2

Letting the contoller C of the original system $S(P,C)$ be selected as in Lemma 1 as well as the reduced system $S(G,C_+)$ be constituted by the unstable part of P and the corresponding stabilizer C_+, then there exist all-parametric controllers C_+ satisfying

$$C_+ = \{(Y_+ + A_{+1}K_+)(X_+ - B_{+1}K_+) \mid K_+ \in \Re, \det(X_+ - B_{+1}K_+) \ne 0\} \tag{15}$$

where

$$G = A_+^{-1}B_+ = B_{+1}A_{+1}^{-1}, \quad \begin{bmatrix} X_{+1} & Y_{+1} \\ -B_+ & A_+ \end{bmatrix}\begin{bmatrix} A_{+1} & -Y_+ \\ B_{+1} & X_+ \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

and

$$(Y_+ + A_{+1}K_+) = (Y + A_1K), \quad (X_+ - B_{+1}K_+) = (X - B_1K) + P_-(Y + A_1K) \tag{16}$$

such that S(P,C) is asymptotically stable if and only if $S(G,C_+)$ is asymptotically stable.

Proof:

IF:

Suppose $S(G,C_+)$ is asymptotically stable, i.e. $H(G,C_+) \in \Re$. Define

$$L \equiv [(X - B_1K) + P(Y + A_1K)]^{-1} \tag{17}$$

Then, it is seen that from (16)

$$L \equiv [(X - B_1K) + (G + P_-)(Y + A_1K)]^{-1}$$

$$=[(X_+ - B_{+1}K_+) + G(Y_+ + A_{+1}K_+)]^{-1} = L_+ \qquad (18)$$

Therefore

$$H(P,C) \equiv \begin{bmatrix} (I+PC)^{-1} & -(I+PC)^{-1}P \\ C(I+PC)^{-1} & I - C(I+PC)^{-1}P \end{bmatrix}$$

$$= \begin{bmatrix} (X - B_1K)L & -(X - B_1K)LP \\ (Y + A_1K)L & I - (Y + A_1K)LP \end{bmatrix}$$

$$= \begin{bmatrix} [(X_+ - B_{+1}K_+) - P_-(Y_+ + A_{+1}K_+)]L_+ & -[(X_+ - B_{+1}K_+) - P_-(Y_+ + A_{+1}K_+)]L_+(G + P_-) \\ (Y_+ + A_{+1}K_+)L_+ & I - (Y_+ + A_{+1}K_+)L_+(G + P_-) \end{bmatrix}$$

$$= H_{+1} + H_{+2}$$

where

$$H_{+1} = \begin{bmatrix} (X_+ - B_{+1}K_+)L_+ & -(X_+ - B_{+1}K_+)L_+G \\ (Y_+ + A_{+1}K_+)L_+ & I - (Y_+ + A_{+1}K_+)L_+G \end{bmatrix} = H(G, C_+)$$

and

$$H_{+2} = \begin{bmatrix} -P_-(Y_+ + A_{+1}K_+)L_+ & P_-(Y_+ + A_{+1}K_+)L_+(G + P_-) - (X_+ - B_{+1}K_+)L_+P_- \\ 0 & -(Y_+ + A_{+1}K_+)L_+P_- \end{bmatrix}$$

$$= \begin{bmatrix} -P_-C_+S_+ & P_-C_+S_+(G + P_-) - S_+P_- \\ 0 & -S_+P_- \end{bmatrix}$$

Since $H(G,C_+) \in \Re$ then, $S_+, S_+G, C_+S_+, C_+S_+G$ and $P_- \in \Re$ also. Thus $H_{+1}, H_{+2} \in \Re$, as is $H(P,C)$.

ONLY IF:

Similarly, suppose $H(P,C) \equiv \begin{bmatrix} S & -SP \\ CS & I - CSP \end{bmatrix} \in \Re$, then

$$H(G,C_+) \equiv \begin{bmatrix} (X_+ - B_{+1}K_+)L_+ & -(X_+ - B_{+1}K_+)L_+G \\ (Y_+ + A_{+1}K_+)L_+ & I - (Y_+ + A_{+1}K_+)L_+G \end{bmatrix}$$

$$= \begin{bmatrix} [(X - B_1K) + P_-(Y + A_1K)]L & -[(X - B_1K) - P_-(Y + A_1K)]L(P - P_-) \\ (Y + A_1K)L & I - (Y + A_1K)L(P - P_-) \end{bmatrix}$$

$$= H_1 + H_2$$

where

$$H_1 = \begin{bmatrix} (X - B_1K)L & -(X - B_1K)LP \\ (Y + A_1K)L & I - (Y + A_1K)LP \end{bmatrix} = H(P,C)$$

and

$$H_2 = \begin{bmatrix} P_-(Y + A_1K)L & -P_-(Y + A_1K)L(P - P_-) + (X - B_1K)LP_- \\ 0 & (Y + A_1K)LP_- \end{bmatrix}$$

$$= \begin{bmatrix} P_-CS & -P_-CS(P - P_-) + SP_- \\ 0 & SP_- \end{bmatrix}$$

Again, since $H(P,C) \in \Re$ then, S, SP, CS, CSP and $P_- \in \Re$, then H_{+1}, $H_{+2} \in \Re$, as is $H(G,C_+)$.

Q.E.D

Lemma 2 provides the technique to stabilize the unstable plant via its intuitive reduced-order model. Analogous to the proposed properties in [5], there are no restrictions on the target plant. It can be unstable, improper, nonminimum phase, MIMO, and even nonrectangular. This result release almost all obstructs in the development of model-reduction control. Moreover, it is utterly imperative for industrial application to compensate the whole system to achieve an acceptable characteristic while it originally doesn' t. In particular, a traditional multipurpose controller [18] is usually needed to achieve the following design objectives:

(i) arbitrary pole assignment, and

(ii) some zero assignments to deal with the problem of disturbance rejection and reference signal tracking.

The important of pole assignment has been shown in many literature, we do not repeat this in here. Reconsider the system of Fig. 1, the output is given by

$$y = Pu$$

And the closed-loop transfer matrix is yielded as

$$T = C(I + PC)^{-1} P$$

Rearrange the sensitivity matrix

$$\begin{aligned} S \equiv (I + PC)^{-1} &= \left[(X - B_1 K)(X - B_1 K)^{-1} + P(Y + A_1 K)(X - B_1 K)^{-1}\right]^{-1} \\ &= (X - B_1 K)\left[(X - B_1 K) + P(Y + A_1 K)\right]^{-1} \\ &= (X - B_1 K)L \end{aligned} \tag{19}$$

Moreover, follow (18) and Lemma 2 we have

$$\begin{aligned} T &= (Y + A_1 K)(X - B_1 K)^{-1}(X - B_1 K)LP \\ &= (Y + A_1 K)LP \\ &= (Y_+ + A_{+1} K_+)L_+(G + P_-) \end{aligned} \tag{20}$$

It is easily seen that the closed-loop transform matrices of these two systems (G and P) have a similar outlook and so do their response behaviors. Therefore one can analyze the over-all behavior or performance from the reduced-order model directly. Since the all-parametric controller synthesize algorithm need to choose a suitable $K \in \Re$ and

$$\begin{aligned} L \equiv \left[(X - B_1 K) + P(Y + A_1 K)\right]^{-1} &= \left[(X - B_1 K) + B_1 A_1^{-1}(Y + A_1 K)\right]^{-1} \\ &= (X + B_1 A_1^{-1} Y)^{-1} \end{aligned} \tag{21}$$

will not be affected by the selection of K and so does P in general, the synthesis problem of arbitrary pole-placement controller can be solved by the selection of suitable poles of $(Y + A_1 K)$, or equivalently, by the selection of suitable poles of $(Y_+ + A_{+1} K_+)$. This implies that one can only take the reduced model to start the analysis and synthesis of the multipurpose controller, and, after obtaining the reduced controller C_+, end the jobs with substituting the desired controller C by Lemma 2. We summarize the design algorithm as in Lemma 3.

Lemma 3

Letting the controller C_+ of the original system S (P, C) be selected as in Lemma 1 as well as the reduced system S (G, C_+) be constituted as in Lemma 2, then the arbitrary pole-placement control can be achieved if there exist an auxiliary function $K_+ \in \Re$ such that

$$(Y_+(s) + A_{+1}(s) K_+(s)) L_+ = Z(s) \tag{22}$$

where $Z(s)$ is an arbitrary matrix function with desired poles.

Remark 2: Lemma 2 and 3 has shown that the analysis and synthesis problem of a system can be solved via its unstable subsystem. Since, in general, the unstable part of a plant only has a smaller dimension than that of the original, and so is the corresponding controller, this new methodology in the above Lemmas would facilitate the design objectivity. Furthermore, if the system is all stable, the intuitively model-reduction control algorithm is still available by starting with an arbitrarily selected subpart.

Now consider the second objective of multipurpose control. The tracking error with disturbance in Fig.1 is defined as

$$e = r - y - d \tag{23}$$

With the sensitivity matrix defined as (1), we have

$$e = S(r - d) \tag{24}$$

and $u = CS(r - d)$

To track and reject high order reference signals $r(t)$ and disturbances $d(t)$ (for example, unit step or ramp signals etc.), the right hand side of eqn. (24) musts not have any pole in RHP [18], i.e. the sensitivity function matrix $S(s)$ must have a large enough number of zeros to cancel the poles of $r(s) - d(s)$ in RHP, where $r(s)$, $d(s)$ denote the Laplace transform of $r(t)$ and $d(t)$ respectively. Similarly, let

$$Q(s) = \begin{bmatrix} r_1(s) - d_1(s) \\ r_2(s) - d_2(s) \\ \dots \\ r_n(s) - d_n(s) \end{bmatrix} \tag{25}$$

where

$$Q_i(s) = \frac{\boldsymbol{h}_i \prod_{k=1}^{r_i} (s - w_{ik})}{\prod_{k=1}^{p_i} (s - u_{ik}) \prod_{k=1}^{q_i} (s - v_{ik})} \qquad \text{for i=1..n,} \tag{26}$$

as $\boldsymbol{h}_i$ is the real gain and u_{ik}, v_{ik}, w_{ik} are the complex poles and zeros. Again, we assume

$$f_i(s) = \prod_{k=1}^{p_i} (s - u_{ik}) \tag{27}$$

denotes the unstable poles of Q_i (s). For the purpose of asymptotically tracking and disturbance rejection, we have to synthesize a controller C to guarantee

$$\lim_{t \to \infty} e(t) = 0$$

i.e., the sensitivity function S has the same nonminimum phase to cancel $f(s)$. Such that

$$e(s) = S(s)Q(s) \in \Re \tag{28}$$

Recalling (19) and (21), the synthesis problem of reference tracking and disturbance rejection controller can be solved by the selection of suitable zeros of $(X - B_1 K) \in \Re$ to achieve pole-zero cancellation with r, d. Moreover, following Lemma 2 we have

$$e = [(X_+ - B_{+1} K_+) - P_-(Y_+ + A_{+1} K_+)] L_+ (r - d)\,. \tag{29}$$

Thus the asymptotically reference tracking and disturbance rejection control algorithm is derived in Lemma 4.

Lemma 4

Letting the contoller C of the original system S (P, C) be selected as in Lemma 1 as well as the reduced system

$S(G,C_+)$ be constituted as in Lemma 2, and the reference and disturbance signals are described as (25)-(29), then the asymptotically output tracking and rejection occurs if there exist an auxiliary function $K_+ \in \Re$ such that

$$\left(X_+(s) - B_{+1}(s)K_+(s)\right) - P_-(s)(Y_+(s) + A_{+1}(s)K_+(s)) = f(s)W(s) \qquad (30)$$

where $f(s)$ is defined by (27) and $W(s)$ is an arbitrary matrix function without unstable mode as $f(s)$.

Remark 3: Since the all-parametric controller ensures the internal stability of systems [5], i.e., there is no unstable pole-zero cancellation occurred between P and C, the stable subsystem P_- just plays the role of another auxiliary transfer matrix in Lemma 4. As the major computational efforts lie in how to solving the factorization pairs in (3), this new technique will still diminish a good deal of the desired works in the design procedure. Notably, if the stable part is of very high order, e.g., $(s+1)^7$, the model-reduction control algorithm is clearly helpful for the analysis and design of systems.

Remark 4: Since the required parameters in lemma 3 are not entirely the same as those in lemma 4, it is obvious that the design criteria of (22) and (30) are linear dependent. Therefore the multipurpose controller is not hard to obtain via a suitable selection of $K_+(s)$. Furthermore, as there is no need to guarantee $K = K_+$ in eqn. (30), the choice of K may also provide some degree of freedoms for performance design. The completely pole-zero assignment or other optimal design is also possible achieved from the identity (20) .

Remark 5: Although the derivations of this section are based on the unstable subsystem, the application of these results to an all-stable system is quite straightforward. For convenience, we sort out the intuitive model-reduction control algorithm as follow:

i) Split the original system into two subsystems. For unstable systems, one of the subsystems must contain all the unstable poles. Take the unstable subsystem (or the lower-order one for stable system) as the reducing-order model.

ii) Follow lemma 2 to find out all parameters, i.e., the A_+, B_+, A_{+1}, B_{+1}, X_+, Y_+, X_{+1} and Y_{+1}.

iii)Follow Lemma 3 and 4 to determine a suitable K_+ for multipurpose control.

iv) According to lemma 2, the desired controller is yielded from eqn. (16) with the second auxiliary parameter K.

An example is now given to confirm the efficiency of this new methodology.

V. An Illustrative Example

For illustration, we consider the system in Fig.1 with a fourth-order unstable and nonminimum-phase transfer function which used in several literature [18,19].

$$P(s) = \frac{60s^3 + 25850s^2 + 685000s - 2500000}{s^4 + 105s^3 + 10450s^2 + 45000s - 500000}$$

Where $P(s)$ have one RHP pole and one RHP zero. Actually, $P(s)$ has poles at $-50 \pm 86.60i$, -10, 5 and zeros at –402.2, -31.89, 3.248. Several authors had attempted to derive different techniques for obtaining reduced model for this unstable plant. All of these techniques will take numerous mathematic or numerical works to find the suitable parameters of $G(s)$. Nevertheless, it cannot guarantee that the controlled behavior of the reduced model is equal to that of the original unstable system. Thus our results of intuitively model reduction control would facilitate the design. The original plant is easily split to

$$P(s) = \left[\frac{s+5}{s-5} + \frac{-s^3 + 60s^2 + 14000s + 600000}{s^3 + 110s^2 + 11000s + 100000}\right]$$

It is straightforwardly to yield the reduced model as

$$G = \frac{s+5}{s-5}$$

Now suppose we have to design a suitable controller to stabilize the closed-loop system in Fig. 1, to assignment a pole in 3s+5=0, and to track an unit step reference input. Utilize the all-parametric controller synthesis algorithm in Lemma 1; we select the left and right coprime matrices of G

$$A_+ = A_{+1} = \frac{s-5}{3s+5}, \qquad B_+ = B_{+1} = \frac{s+5}{3s+5}$$

and the Bezout identity pairs

$$X_+ = X_{+1} = 1, \qquad Y_+ = Y_{+1} = 2$$

Such that

$$X_+ A_+ + Y_+ B_+ = 1$$

and the all-parametric stabilizer is yield as (15)

$$C_+ = (Y_+ + A_+ K)(X_+ - B_+ K)^{-1}$$

Moreover, as the system needs to track a unit step reference signal, K_+ is determined from

$$\left(X_+(s) - B_{+1}(s)K_+(s)\right) - P_-(s)(Y_+(s) + A_{+1}(s)K_+(s))$$

has a zero at $s = 0$ to cancel the unstable mode of reference input. And

$$(Y_+(s) + A_{+1}(s)K_+(s))L_+$$

contains the pole at $s = -5/3$, Thus

$$K_+ = 2.2$$

At last, according to Lemma 2, it is easily to yield the desired multipurpose controller with $K = K_+$ .and

$$P_- = \frac{-s^3 + 60s^2 + 14000s + 600000}{s^3 + 110s^2 + 11000s + 100000}$$

$$C(s) = \frac{8.2s^4 + 901s^3 + 90090s^2 - 809000s - 100000}{9s^4 + 573s^3 - 106720s^2 + 4892000s}$$

Such that the closed-loop transfer function has poles at –5/3, -10, and -50$\pm j86.6$.

Figure 2 demonstrates the asymptotically stable and reference signal tracking properties of the closed-loop system. In this example, it is seen that the synthesis of multipurpose controller for this rather high-order system has been reduced to a simple case, and so that X, Y, K can be selected as simple scalars. This really diminishes a good deal of the desired efforts in the design procedure.

VI. Conclusions

In this paper, the extended all-parametric controller synthesize algorithm via a reduction-order model is derived. Based on this new methodology, intuitive model-reduction control problem can be easily solved without any restriction on the plant as those in the previous literature. That is, the system can be unstable, improper, nonminimum phase, MIMO, and even nonrectangular. While one is not only needed to find an approximation model but also needed to compensate characteristics of the whole system, this new technique possesses much computational advantage over the existing approaches in multipurpose control.

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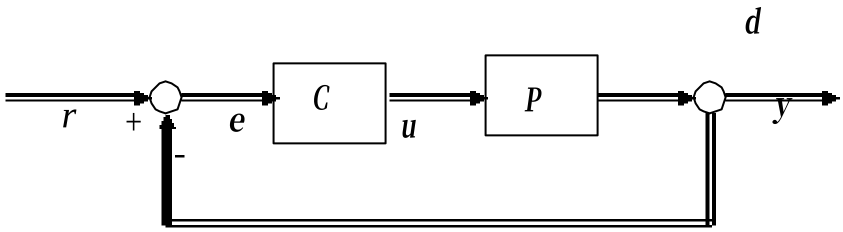

Fig 1 The closed-loop system.

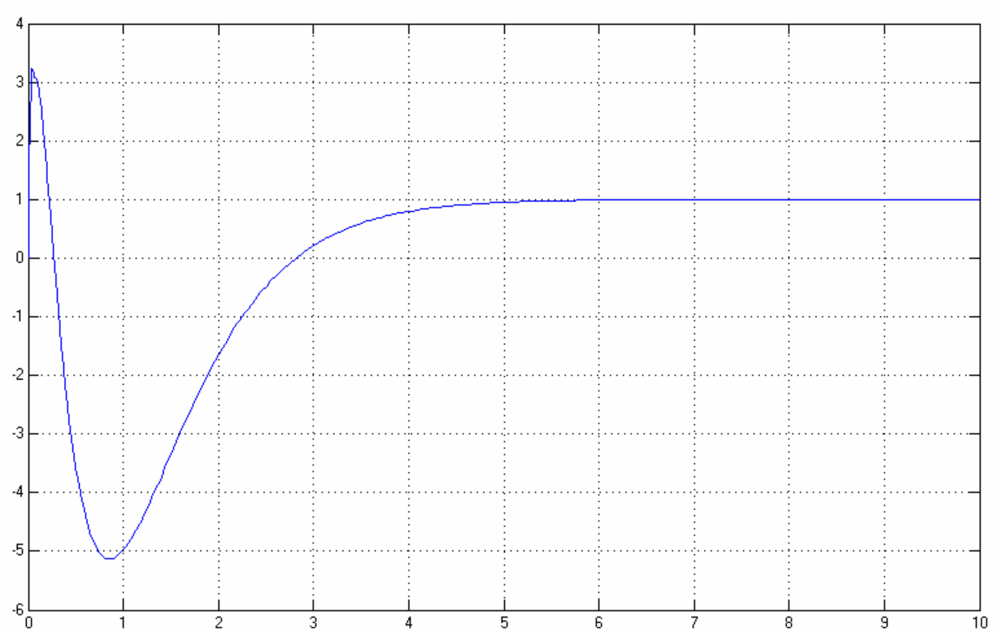

Fig 2 Step response of the example.