國　立　宜　蘭　大　學

96 學年度轉學招生考試

（考生填寫）
准考證號碼：

微　積　分　試　題

《作答注意事項》

1. 請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
2. 考試時間：80 分鐘。
3. 本試卷共有 15 題單選題，1-10 題每題 6 分，11-15 題每題 8 分，共計 100 分，答錯不倒扣。
4. 請將答案寫在答案卷上。（請用黑、藍原子筆作答）
5. 考試中禁止使用大哥大或其他通信設備。
6. 考試後，請將試題卷及答案卷一併繳交。
7. 本試卷採雙面影印，請勿漏答。

1. $\lim_{x\to 1}\frac{|x^3+x^2+x-4|-1}{x-1}=$ (A) 1 (B) -6 (C) 0 (D) -2 Hint: $|a|=-a$ if $a<0$

2. If $h(x)=\sin^2\sqrt{x}$, then $h'(x)=$ (A) $2(\sin\sqrt{x})(\cos\sqrt{x})(\frac{1}{2\sqrt{x}})$ (B) $2(\cos\sqrt{x})(\frac{1}{2}x^{\frac{-1}{2}})$
(C) $2(\sin\sqrt{x})(\frac{1}{2\sqrt{x}})$ (D) $2\cos\frac{1}{2\sqrt{x}}$.

   Hint: Chain rule: $(f\circ g)'(x)=\underline{f'(g(x))}\cdot\underline{g'(x)}$ or $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$ if $x\to u\to y$

3. $\int_0^2\frac{1}{4+x^2}dx=$ (A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$ Hint: $(\tan^{-1}x)'=\frac{1}{1+x^2}$

4. If $f(x,y)=x^2-y^2+2x-3y-1$, find the directional derivative $D_{\vec{u}}f(P)$ at the point $P(1,1)$ in the direction $\vec{u}=(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$. (A) $\frac{-\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\sqrt{2}$ (D) $-\sqrt{2}$.

   Hint: $D_{\vec{u}}f(P)=\nabla f(x,y)|_P\bullet\vec{u}$

5. $\int_0^\infty xe^{-x}dx=$ (A) ln2 (B) e (C) 1 (D) ∞

   Hint: Integration by parts: $\int udv=uv-\int vdu$

6. The function $y=f(x)=x^3+ax^2+bx+c$ has neither relative maximum nor relative minimum if and only if (A) $b^2-4ac\geq 0$ (B) $b^2-4ac\leq 0$ (C) $a^2-3b\leq 0$ (D) $a^2-3b\geq 0$. Hint: $f'(x)\geq 0$ for every $x\in R$

7. If $x^2y=1, x>0, y>0$, find the minimum value of x^2+4xy. (A) $3\cdot 2^{\frac{2}{3}}$ (B) $2\cdot 3^{\frac{1}{3}}$
(C) $2\cdot 2^{\frac{1}{3}}$ (D) $3\cdot 3^{\frac{1}{2}}$ Hint: Consider the function $f(x)=x^2+\frac{4}{x}$, $x>0$.

8. If $F(x)=\int_0^x(\int_0^t\exp s^2ds)dt$, then $F''(x)=$ (A) $2\exp x$ (B) $\exp x^2$ (C) $2x\exp x^2$
(D) $\frac{1}{3}\exp x^3$. Hint: $\frac{d(\int_a^x f(t)dt)}{dx}=f(x)$

9. Find an equation of the tangent plane to the graph of the equation $z=xy$ at the point $P(2,-3,-6)$.
(A) $2(x-2)-3(y+3)-(z+6)=0$ (B) $-3(x-2)+2(y+3)+(z+6)=0$
(C) $2(x-2)-3(y+3)+(z+6)=0$ (D) $-3(x-2)+2(y+3)-(z+6)=0$

   Hint: A normal vector $\vec{N}$ of the tangent plane at the point P is given by $(\frac{\partial z}{\partial x},\frac{\partial z}{\partial y},-1)|_P$.

背面尚有試題

10. Find the curvature κ of the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ at the point (2,0). (A) 4 (B) 1

(C) $\frac{1}{2}$ (D) 2 Hint: Let $x=2\cos t, y=\sin t$ and $\kappa=\frac{|\vec{v}\times\vec{a}|}{|\vec{v}|^3}$ at $t=0$.

11. Let S be the ellipsoid $\frac{x^2}{1}+\frac{y^2}{4}+\frac{z^2}{9}\le 1$, then the triple integral $\iiint_S dxdydz=$ (A) 12π

(B) 16π (C) 6π (D) 8π. Hint: The transformation $x=u$, $y=2v$, $z=3w$ maps the sphere $u^2+v^2+w^2\le 1$ into S and its Jacobian $J(u,v,w)=6$.

12. $\lim_{n\to\infty}(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+2n})=$ (A) $2\ln 3$ (B) $2\ln 2$ (C) $\ln 3$ (D) $\ln 2$

Hint: $\int_n^m \frac{1}{t}dt=\ln m-\ln n=\ln\frac{m}{n}$ if n, m are positive integers.

13. Evaluate the line integral $\oint_\lambda \frac{ydx-xdy}{x^2+y^2}$ where λ is the circle $x^2+y^2=4$ oriented counterclockwise. (A) 2π (B) -2π (C) 4π (D) -4π

Hint: Letting $x=2\cos t, y=2\sin t$, $dx=-2\sin t dt$, $dy=2\cos t dt$

14. If $|x|<1$ then $\ln(1+x)=$ (A) $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$ (B) $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$

(C) $x-\frac{x^2}{2!}+\frac{x^3}{3!}-\frac{x^4}{4!}+\cdots$ (D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$. Hint: $D\ln(1+x)=\frac{1}{1+x}$

15. Which function is continuous at the origin $(0,0)$?

(A) $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ (B) $f(x,y)=\begin{cases}\frac{xy^2}{x^2+y^4} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$

(C) $f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ (D) $f(x,y)=\begin{cases}\frac{\sin(x^2+y^2)}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$

Hint: $f(x,y)$ is said to be continuous at the point (x_0,y_0) if for every point sequence (x_1,y_1), (x_2,y_2), $(x_3,y_3)\cdots\to(x_0,y_0)$, we have $f(x_1,y_1)$, $f(x_2,y_2)$, $f(x_3,y_3)\cdots\to f(x_0,y_0)$.

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