

國立宜蘭大學

105 學年度轉學招生考試

(考生填寫)

准考證號碼：

工程數學(僅考微分方程)試題

《作答注意事項》

- 1.請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
- 2.考試時間：80 分鐘。
- 3.本試卷選擇題 40 分，非選擇題 60 分，共計 100 分。
- 4.請將答案寫在答案卷上（於本試題上作答者，不予計分）。
- 5.考試中禁止使用手機或其他通信設備。
- 6.考試後，請將試題卷及答案卷一併繳交。
- 7.本試卷採雙面影印，請勿漏答。
- 8.應試時不得使用電子計算機。

I. 選擇題(40%)

1. () Please identify linear of following differential equation and verify that the given function is the solution.(a,b,c are arbitrary constant).

(A) $y' + 2y = 4(x^2 + 1)^2$, $y = e^{3x} + 2x^2 + 2x + 1$ (B) $y'^2 = -\frac{y}{x}$, $xy = c$

(C) $y' + \frac{x^2 - y^2}{2xy} = 0$, $y^2 = ax - x^2$ (D) $(y')^2 - xy' + y = 0$, $y = cx - c^2$

(E) $y' + 4y = 2$, $y = ce^{-4x} + x^2 - 2x$ (10%)單選

2. () Solve the O.D.E (Euler Cauchy equation). $x^3 y^{(3)} - 3x^2 y'' + 6xy' - 12y = 0$

(A) $y = C_1 x^4 + x [C_2 \cos(\sqrt{2} \ln x) + C_3 \sin(\sqrt{2} \ln x)]$

(B) $y = C_1 e^{4x} + e^x [C_2 \cos(\sqrt{2}x) + C_3 \sin(\sqrt{2}x)]$

(C) $y = C_1 e^{-4x} + e^x [C_2 \cos(\sqrt{2}x) + C_3 \sin(\sqrt{2}x)]$

(D) $y = C_1 x^{-4} + x [C_2 \cos(\sqrt{2} \ln x) + C_3 \sin(\sqrt{2} \ln x)]$

(E) $y = C_1 x^{-4} + x [C_2 \cos(\sqrt{2}x) + C_3 \sin(\sqrt{2}x)]$ (10%)單選

3. () Solve the differential equation : $y''' + 3y'' + 3y' + y = 30e^{-x}$

(A) $y = c_1 e^x + c_2 x e^x + c_3 x^2 e^x + 5x^3 e^{-x}$ (B) $y = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x} + 5x^3 e^{-x}$

(C) $y = c_1 e^{-3x} + c_2 x e^{-3x} + c_3 x^2 e^{-3x} + 5x^3 e^{-x}$ (D) $y = c_1 e^{-2x} + c_2 x e^{-3x} + c_3 x^2 e^{-3x} + 5x^3 e^{-x}$

(E) $y = c_1 e^x + c_2 x e^x + c_3 x^2 e^x + 5x^3 e^{-x}$ (10%)單選

4. () Please solve the differential equation : $y'' + y = \sec x$

(A) $y = c_1 \cos x + c_2 \sin x + \cos x \ln|\cos x| + x \sin x$

(B) $y = c_1 \cos x + c_2 \sin x + \cos x \ln|\sec x| + x \csc x$

(C) $y = c_1 \cos x + c_2 \sin x + \cos x \ln|\sin x| + x \cos x$

(D) $y = c_1 \cos x + c_2 \sin x + \sec x \ln|\cos x| + x \sin x$

(E) $y = c_1 \cos x + c_2 \sin x + x \ln|\cos x| + x \sin x$ (10%)單選

II. 非選擇題(60%)

1. Solve the following differential equation.

$$y'' + 4y' + 3y = 0, \quad y(0) = 0, \quad y'(0) = 4 \quad (10\%)$$

2. Solve the differential equation : $\tilde{y}' = \tilde{A}\tilde{y} = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \tilde{y}$, $\tilde{y} = \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix}$ (10%)

3. Please find the function $y(x)$ to satisfy the differential equation $y'' + 2y' + 10y = 0$. (10%)

4. Please find the function $y(x)$ to satisfy the equation and initial conditions.

$$ydy = 4x(y^2 + 1)^{\frac{1}{2}} dx, \quad y(0) = 1 \quad (10\%)$$

5. Solve $xy' + y = 0$ in general solution (10%)

6. Find the solution $u(x, y)$ of $y^2 u_{yy} + 2yu_y - 2u = 0$,

$$u(0, y) = 0, u(1, y) = \frac{y^3 + 4}{y^2}, u(x, 1) = 5x, u(x, -1) = 3x. \quad (10\%)$$