國 立 宜 蘭 大 學

100 學年度轉學招生考試

（考生填寫）
准考證號碼：

微 積 分 試 題

《作答注意事項》

1. 請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
2. 考試時間：80 分鐘。
3. 本試卷共有單選擇題 20 題，一題 5 分，不倒扣，共計 100 分。
4. 請將答案寫在答案卷上。
5. 考試中禁止使用大哥大或其他通信設備。
6. 考試後，請將試題卷及答案卷一併繳交。
7. 本試卷採雙面影印，請勿漏答。

1. Let $f(x)=x^3$, $P=\left\{-1,\frac{-2}{3},\frac{-1}{3},0\right\}$ be a partition of $[-1,0]$. Find the lower sum $S(P,f)$, and the upper sum $T(P,f)$. (A) $\frac{4}{9},\frac{1}{9}$ (B) $\frac{4}{3},\frac{1}{3}$ (C) $\frac{-4}{3},\frac{-1}{3}$ (D) $\frac{-4}{9},\frac{-1}{9}$

2. Let $[x]$ be the smallest integer $\le x$, then $\int_2^3 [x]\,dx$ =(A) 3 (B) 2 (C) 1 (D) 0

3. $\int x^5(x^6+2)^{-2}dx$ =(A) $\frac{-1}{6}(x^6+2)^{-1}+c$ (B) $(x^6+2)^{-1}+c$ (C) $\frac{-1}{6}(x^6+2)^{-3}+c$ (D) $(x^6+2)^{-3}+c$

4. $\int_{-2}^{2}|x|dx$ =(A) 0 (B) 1 (C) 2 (D) 4

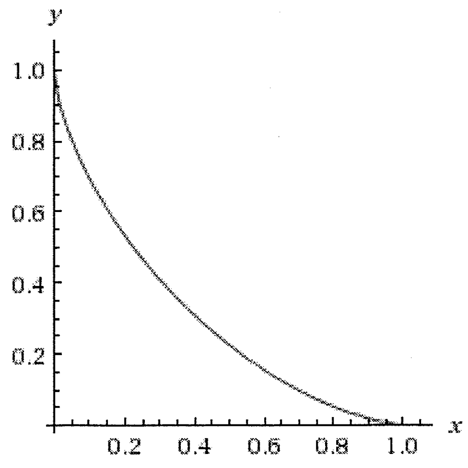
5. $\int_0^{2\pi}\cos\frac{x}{4}\,dx$ =(A) 4 (B) 2 (C) 1 (D) 0

6. $\int(\sin^2 \underline{3x})dx$ =(A) $\frac{1}{2}(x-\sin 6x)+c$ (B) $\frac{1}{2}(x-\frac{1}{6}\sin x)+c$ (C) $\frac{1}{2}(x-\frac{1}{6}\sin 6x)+c$ (D) $\frac{1}{2}(x-\sin\frac{x}{6})+c$

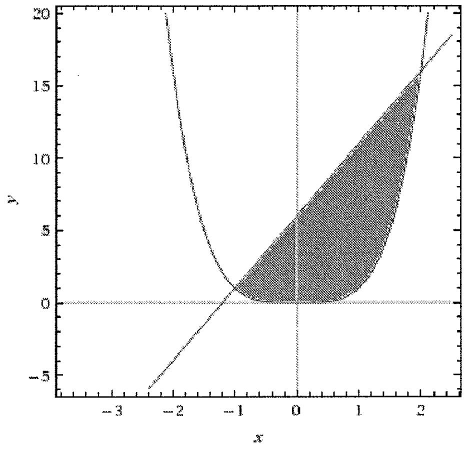
7. If $f(x)=\sqrt{x}$, find a number $c\in(0,1)$ such that $\int_0^1 f(x)dx=f(c)(1-0)$. c =(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) $\frac{2}{9}$ (D) $\frac{4}{9}$

8. Find $\int_0^1 x^2dx$ =(A) $\lim_{n\to\infty}(\sum_{i=1}^{n}(\frac{i}{n}\cdot\frac{1}{n}))$ (B) $\lim_{n\to\infty}(\sum_{i=1}^{n}(\frac{i}{n})^2\cdot(\frac{1}{n}))$ (C) $\lim_{n\to\infty}(\sum_{i=1}^{n}(\frac{i}{n})^2\cdot(\frac{1}{n})^2)$ (D) $\lim_{n\to\infty}(\sum_{i=1}^{n}\frac{i}{n}\cdot(\frac{1}{n})^2)$

9. Find the arc length of the graph of $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$ in the first quadrant. (A) $\frac{3}{2}$ (B) $\frac{4}{3}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$



10. Find the area of the region bounded above by the graph of $y = x^4$ and down by the line passing through the points (-1,1) and (2,16). (A) 18.1 (B) 18.3 (C) 18.6 (D) 18.9



11. Is the function defined by $z = f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ continuous at $(0,0)$?

(A) Yes. (B) No. (C) It depends. (D) The hypothesis is not sufficient.

12. If $F(x,y) = \tan^{-1}\dfrac{y}{x}$, then $\dfrac{\partial^2 F}{\partial x^2} + \dfrac{\partial^2 F}{\partial y^2} =$ (A) -1 (B) 0 (C) 1 (D) $\dfrac{y}{x}$.

13. If $f(x,y) = xy$, find the directional derivative $D_{\vec{u}} f(P)$ at the point P=(2,2) in the direction of the unit vector $\vec{u} = (\cos\dfrac{\pi}{4}, \sin\dfrac{\pi}{4})$. (A) $\sqrt{2}$ (B) 2 (C) 1 (D) $2\sqrt{2}$

14. Find the equation of the tangent plane to the graph of the function $z = f(x,y) = \sqrt{x^2+y^2}$ at the point $P(3,4,5)$.

(A) $3(x-3)+4(y-4)+5(z-5)=0$ (B) $3(x-3)-4(y-4)-5(z-5)=0$

(C) $3(x-3)+4(y-4)-5(z-5)=0$ (D) $3(x-3)-4(y-4)+5(z-5)=0$

15. If $z = f(x,y)$, and $x = r\cos\theta$, $y = r\sin\theta$, then $(\dfrac{\partial z}{\partial r})^2 + \dfrac{1}{r^2}(\dfrac{\partial z}{\partial \theta})^2 =$

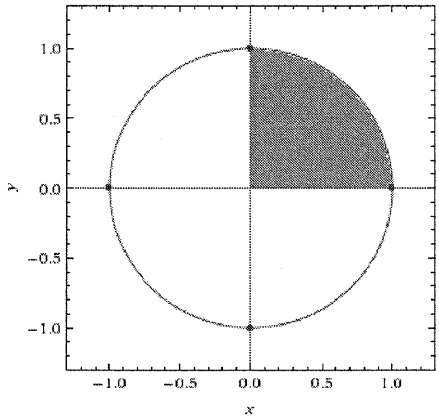
(A) $(\dfrac{\partial z}{\partial x})^2 + (\dfrac{\partial z}{\partial y})^2$ (B) $\dfrac{\partial z}{\partial x} \cdot \dfrac{\partial z}{\partial y}$ (C) 1 (D)2

16. If $Q = \{(x,y) | 0 \le x \le 1, 0 \le y \le 2\}$, find $\int_Q xy\, dA$.(A) 4 (B) 2 (C) 1 (D) 0

17. $\int_0^1 (\int_{\frac{y}{2}}^{\frac{1}{2}} e^{-x^2} dx) dy =$(A) $-(e^{\frac{-1}{4}} - 1)$ (B) $-(e^{\frac{-1}{3}} - 1)$ (C) $-(e^{\frac{-1}{2}} - 1)$ (D) $-(e^{-1} - 1)$

18. Find the center of mass of the homogenous lamia $Q = \{(x,y) | x^2+y^2 \le 1, x \ge 0, y \ge 0\}$,

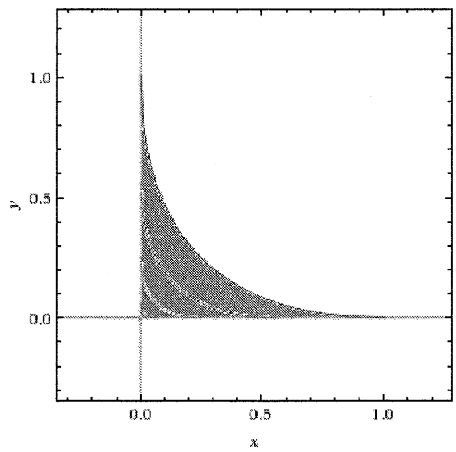
which is one quarter of a unit disc. (A) $(\frac{1}{2},\frac{1}{2})$ (B) $(\frac{4}{3\pi},\frac{4}{3\pi})$ (C) $(\frac{2}{3},\frac{2}{3})$ (D) $(\frac{3}{2\pi},\frac{3}{2\pi})$



19. Find the volume of the solid bounded by $\sqrt{x}+\sqrt{y}+\sqrt{z}=1$ and the coordinate planes.

(A) $\int_0^1\int_0^{(1-\sqrt{x})}(1-\sqrt{x}-\sqrt{y})^2\,dydx$ (B) $\int_0^1\int_0^{(1-\sqrt{x})^2}(1-\sqrt{x}-\sqrt{y})\,dydx$

(C) $\int_0^1\int_0^{(1-\sqrt{x})^2}(1-\sqrt{x}-\sqrt{y})^2\,dydx$ (D) $\int_0^1\int_0^{(1-\sqrt{x})}(1-\sqrt{x}-\sqrt{y})\,dydx$



20. Find the center of mass of the homogenous solid

$S=\left\{(x,y,z)\middle|x^2+y^2+z^2\le 1, x\ge 0, y\ge 0, z\ge 0\right\}$, which is one eighth of a unit solid sphere.

(D) $(\frac{9}{8\pi},\frac{9}{8\pi},\frac{9}{8\pi})$ (D) $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ (D) $(\frac{4}{3\pi},\frac{4}{3\pi},\frac{4}{3\pi})$ (D) $(\frac{3}{8},\frac{3}{8},\frac{3}{8})$