九十八學年度研究所碩士班考試入學
機械與機電工程學系碩士班
工程數學考科

1. Try to find the general solution of the following initial value problem.

$$y'+2y\sin 2x = 2e^{\cos 2x} \quad , y(0)=0$$

2. Consider the system described by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t),$$

where $x(t) = [x_1(t) \quad x_2(t)]^T$.

(a) Compute the state transition matrix $\Phi(t,0)$.

(b) Using the state transition matrix from (a) and for the initial conditions $x_1(0) = 1$ and $x_2(0) = -1$, find the solution $x(t)$ for $t \geq 0$.

3. 設 $\mathbf{A}=\begin{bmatrix} 2 & 4 & -6 \\ 4 & 2 & -6 \\ -6 & -6 & -15 \end{bmatrix}$，試將 $\mathbf{v}=[1 \quad 2 \quad -3]^T$ 表示成 A 之所有特徵向量的線性組合。

4. A weight of mass 0.50 kg (weight = 4.9 N) stretches a spring 0.70 m. The weight is pushed 0.40 m above the equilibrium position and released. Find the position of the weight as a function of time, if a damping force numerically equal to twice the velocity is present.

5. The thermal conductivity K of a material, in general, depend on the temperature T (and therefore x), and thus it cannot be taken out of the derivative. However, the thermal conductivity in most applications can be assumed to remain constant at some value. The equation above in that case reduces to:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Where $\dot{e}_{gen}$ is the heat generation of the system, K is constant thermal conductivity, α thermal diffusivity.

When the system is under the steady state, no heat generation condition, please (1) find the solution of the govern equation and (2) discussion with temperature variation for heat conduction of system.