九十八學年度研究所碩士班考試入學
環境工程學系碩士班甲組
工程數學考科

1. **Matrix multiplication**: Please try to solve the solutions of $\mathbf{A}^T\mathbf{B}$ and $\mathbf{B}^T\mathbf{A}$. (<u>10%</u>)

$$\mathbf{A} = \begin{bmatrix} 6 & -2 & -2 \\ 10 & -3 & 1 \\ -10 & 5 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$$

2. You have to solve the following ODE by using the **Power Series Method**, and you only expand to x^4 term. (<u>10%</u>)

$$y''(x) - e^x y(x) = 1 + \sin(x)$$

[**Hint:** Taylor's series; analytic center $x_0=0$; constant c_1, c_2]

3. You have to solve the following ODE by using the **Laplace Transform Method**. (15%)

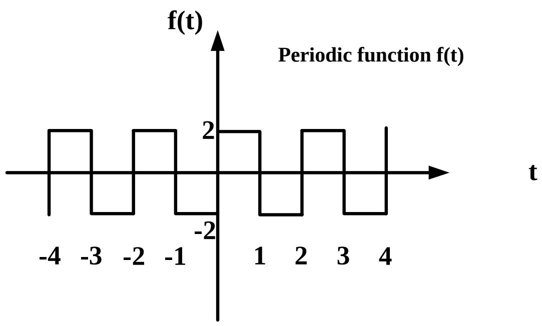
$$y'(t) + 2y(t) + \int_0^t y(x)dx = \sin(t) \text{ and } y(0) = 0$$

[**Hint:** $\mathcal{L}[f'(t)]$; $\mathcal{L}\left[\int_0^t f(x)dx\right]$; $\mathcal{L}[f(t)e^{at}]$]

4. Please try to solve the following ODE by using the **Fourier Series Method,** and the f(t) is a periodic function. (15%)

$$y''(t) + ky(t) = f(t)$$

[**Hint:** Let $y(t) = \sum_{n=1}^{\infty} a_n \sin(n\pi t)$]



5. (1) Please try to solve the following PDE. (15%)

$$\frac{\partial^2 u(x,y)}{\partial x^2} - \frac{\partial^2 u(x,y)}{\partial y^2} = y + \sin(2x+3y) + e^{2x+3y}$$

[**Hint:** Let $u_H(x,y)=f(y+mx)$; $u(x,y)=u_H(x,y)+u_P(x,y)$]

(2) Please try to solve the following one-dimension heat equation by using the **Separating Variable Method**, and α is a constant (20%) [**Hint:** Let $u(x,t)=X(x)T(t)$]

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u(x,t)}{\partial t}$$

B.C. $u(-5,t)=0$ and $u(5,t)=0$ I.C. $u(x,0) = \dfrac{3\pi}{4}$

6. If we can simplify the one-dimension Streeter-Phelps water quality model as follows:

CBOD model: L(t): CBOD at t
k_1: 祛氧係數(constant)

$$0 = -\frac{dL(t)}{dt} - k_1 L(t) \text{ and } L(0) = L_0$$

DO model: C(t): 溶氧 at t
C_s: 飽和溶氧(constant)
k_2: 再曝氣係數 (constant)

$$0 = -\frac{dC(t)}{dt} + k_2(C_s - C(t)) - k_1 L(t)$$

Let D(t)=缺氧量at t=Cs-C(t) and $D(0)=D_0$

Please try to solve the D(t) by using the above-mentioned two ODE models. (15%)