

1. (10%) Find a complete solution of the equation $y'' - 2y' + y = xe^x - e^x$.
2. (10%) Find the general solution of $xdy - (4x^2 + y^2)dy = ydx$.
3. (10%) Prove that if A is an invertible matrix of order n , then $\text{adj}(A^{-1}) = [\text{adj}(A)]^{-1}$.
4. (20%) For $W = \{2s - t, s, t, s\}$, find
 - (a) the basis of W . (10%)
 - (b) the orthogonal complement of W . (10%)
5. (10%) Use power series method to solve ODE $y'' + \cos(x)y = e^x$.
When the solution of problem above is specified as
$$y(x) = \sum_{n=0}^{\infty} a_n x^n = a_0(A + Bx^2 + \cdots) + a_1(Cx + Dx^3 + \cdots) + (Ex^2 + Fx^3 + \cdots), |x| < \infty,$$
evaluate the value of $K = A + B + C + D + E + F = ?$ please.
6. (10%) Find the inverse Laplace transform of the given function: $F(s) = \frac{12s - 24}{(s^2 - 4s + 40)^2}$.
7. (10%) Apply the Fourier transform to solve the following differential equation
$$y' - 4y = \begin{cases} e^{-4t} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$
8. (10%) Find the work W done by the force $\vec{F} = y^2\vec{i} + 2(xy + z)\vec{j} + 2y\vec{k}$ in the displacement of a particle along the straight segment C from $P(0,0,0)$ to $Q(1,1,1)$.
9. (10%) Evaluate $\int_{-i}^i |z| dz = ?$
 - (a) Integrating along a straight line segment. (5%)
 - (b) Integrating along the left half of the unit circle. (5%)