國　立　宜　蘭　大　學

95 學年度轉學招生考試

（考生填寫）
准考證號碼：

微　積　分　試　題

《作答注意事項》

1. 請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
2. 考試時間：80 分鐘。
3. 本試卷共有 15 題單選題，1-10 題每題 6 分，11-15 題每題 8 分，共計 100 分，答錯不倒扣。
4. 請將答案寫在答案卷上。（請用黑、藍原子筆作答）
5. 考試中禁止使用大哥大或其他通信設備。
6. 考試後，請將試題卷及答案卷一併繳交。
7. 本試卷採雙面影印，請勿漏答。

1. $\int_0^{\pi} \sin\frac{x}{2}dx =$ (A) -1 (B) 0 (C) 1 (D)2

2. $\int_2^{e} \frac{dx}{x\ln x} =$(A) $-\ln(\ln 2)$ (B) $-\ln 2$ (C) $\frac{1}{2e}$ (D) $\frac{2}{e}$

   Hint: Let $u = \ln x$

3. $\int_0^{\frac{\pi}{2}} x\sin x dx$ =(A) 1 (B) π (C) 2 (D)$\frac{\pi}{2}$ Hint: $\int u dv = uv - \int v du$

4. $\int_0^{\frac{\pi}{2}} \sin^2(2x)dx$=(A) 1 (B) $\frac{\pi}{4}$ (C) 2 (D)$\frac{\pi}{2}$ Hint: $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$

5. Let $f(x) = \int_0^{x^2} \sin^6 t dt$, then $f'(x) =$ (A) $\sin^6 x^2$ (B) $2x\sin^6 x^2$

   (C) $6\sin^5 x^2$ (D) $6(\sin^5 x^2)\cos x$

6. Let $g(x) = \int_0^x (\int_0^t \sin^6 u du)dt$, then $g''(x) =$ (A) $\sin^6 x$ (B) $2x\sin^6 x$

   (C) $6\sin^5 x$ (D) $6(\sin^5 x)\cos x$

7. The curvature κ of the curve $y = x^2$ at the origin (0,0) is (A) 0 (B) 1 (C) 2 (D)$\frac{1}{2}$

   Hint: (i) $\vec{r}(t) = (t, t^2) \Rightarrow \vec{v}(t) = (1, 2t)$, $\vec{a}(t) = (0,2)$ (ii) $\kappa = \frac{|\vec{v}\times\vec{a}|}{|\vec{v}|^3}$

8. Let S be the solid ball $x^2 + y^2 + z^2 \le 1$, then the triple integral

   $\iiint_S (x^2 + y^2 + z^2)dxdydz$ =(A) $\frac{3\pi}{5}$ (B) $\frac{4\pi}{5}$ (C) $\frac{5\pi}{3}$ (D) $\frac{4\pi}{3}$

   Hint: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2 + y^2 + z^2 = \rho^2$,
   $dxdydz = \rho^2 \sin\phi \, d\theta \, d\phi \, d\rho$

9. Let Ω be the disc $x^2 + y^2 \le 1$, then the double integral $\iint_\Omega (x^2 + y^2)dxdy =$

   (A) π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

   Hint: $x = r\cos\theta$, $y = r\sin\theta$, $x^2 + y^2 = r^2$, $dxdy = rdrd\theta$

10. Let $f(x) = \begin{cases} e^{\frac{-1}{x^2}}, x \ne 0 \\ 0, x = 0 \end{cases}$, then $f'(0) =$ (A)1 (B) e^{-1} (C) 0 (D) $\ln 2$

    Hint: (i) $f'(0) = \lim_{x\to 0}\frac{f(x) - f(0)}{x - 0}$ (ii) $\frac{e^{\frac{-1}{x^2}}}{x} = ye^{-y^2} = \frac{y}{e^{y^2}}$

背面尚有試題

11. $\lim_{n\to\infty}(\sum_{i=1}^{n}\frac{n}{i^2+n^2})$ =(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 1 (D) $\frac{1}{2}$

Hint: $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{n}{i^2+n^2}=\lim_{n\to\infty}\sum_{i=1}^{n}(\frac{n^2}{i^2+n^2})(\frac{1}{n})=\lim_{n\to\infty}\sum_{i=1}^{n}(\frac{1}{(\frac{i}{n})^2+1})(\frac{1}{n})=\int_0^1\frac{1}{x^2+1}dx=$

12. The maximum in the set $\left\{\frac{x}{\sqrt{(x-4)^2+9}}\middle|x>0\right\}$ is (A) $\frac{5}{3}$ (B) $\frac{3}{2}$ (C) $\frac{4}{3}$ (D) 2

Hint: Let $f(x)=\frac{x}{\sqrt{(x-4)^2+9}}$ and solve $f'(x)=0 \Rightarrow (x-4)^2+9=x(x-4)$

13. $\lim_{x\to\infty}(\sqrt[5]{x^5+2x^4+x^3}-x)$ = (A)0 (B) $\frac{1}{5}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

Hint: $\sqrt[5]{x^5+2x^4+x^3}-x=x(\sqrt[5]{1+\frac{2}{x}+\frac{1}{x^2}}-1)=\frac{(1+2x^{-1}+x^{-2})^{\frac{1}{5}}-1}{x^{-1}}=\frac{(1+2y+y^2)^{\frac{1}{5}}-1}{y}$

14. Let $f:[0,1]\to[1,3]$ be a one to one increasing function defined by $f(x)=x^2+x+1$. If $g:[1,3]\to[0,1]$ is the inverse of f i.e. $(f\circ g)(y)=y$, $(g\circ f)(x)=x$, then $\int_1^3 g(y)dy=$

(A) $\frac{4}{3}$ (B) $\frac{5}{3}$ (C) $\frac{7}{6}$ (D) $\frac{5}{6}$

Hint: Let $y=f(x)\Rightarrow dy=f'(x)dx=(2x+1)dx$

15. Let Ω be the region bounded by the four lines $x+y=1, x+y=3, x-y=0, x-y=4$, then the double integral $\iint_\Omega (x^2-y^2)dxdy$ =(A)6 (B) 8 (C)12 (D) 16

Hint: (i) $u=x+y,\ v=x-y \Rightarrow x=\frac{1}{2}(u+v),\ y=\frac{1}{2}(u-v)$

(ii) $J(u,v)=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial y}{\partial u}\\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v}\end{vmatrix}$

(iii) $\iint_\Omega (x^2-y^2)dxdy=\iint_\Gamma uv|J(u,v)|dudv=\int_1^3\int_0^4 uv|J(u,v)|dvdu=.$

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