

Part I. 單擇題 (每題五分、答錯倒扣一分)

1. () The general solution of $y''-2y'+y=12e^x/x^3$ is (A) $(c_1+c_2x)e^{-x}+\frac{3e^x}{x^2}$ (B) $(c_1+c_2x)e^{-x}+\frac{e^x}{x^3}$ (C) $(c_1+c_2x)e^x+\frac{6e^x}{x^3}$ (D) $(c_1+c_2x)e^x+\frac{6e^x}{x}$ (E) $(c_1-c_2x)e^x+\frac{3e^x}{x^2}$.
2. () Use Fourier integral to evaluate the integral $\int_0^\infty \frac{\cos \omega x}{k^2+\omega^2} d\omega$ ($x>0, k>0$) and the result is (A) $\frac{\pi}{2}e^{-kx}$ (B) $\frac{\pi}{2k}e^{-kx}$ (C) $\frac{1}{2}e^{-x}$ (D) $\frac{\pi}{2k}e^{-x}$ (E) $\frac{1}{2}e^{-kx}$.
3. () The Laplace transform of $f(t)=e^{-2t}+\sin t$ is (A) $\frac{s^2+s+1}{s^3+s^2+s+1}$ (B) $\frac{s^2-s+2}{s^3-2s^2+3s+4}$ (C) $\frac{s^2+s+3}{s^3+2s^2+s+2}$ (D) $\frac{s^2-s+4}{s^3-2s^2+2s-1}$ (E) $\frac{s^2+s+5}{s^3+s^2+2s+2}$.
4. () Given that $\mathbf{A}=\begin{bmatrix} 1 & 2 & 3 & -4 & 0 \\ 2 & -1 & 3 & 1 & 2 \\ -2 & 2 & -2 & 1 & -1 \end{bmatrix}$, the rank of \mathbf{A} equals (A)1 (B)2 (C)3 (D)4 (E)5.
5. () A square matrix \mathbf{A} is transformed into another matrix \mathbf{B} by $\mathbf{B}=\mathbf{C}^{-1}\mathbf{A}\mathbf{C}$, where \mathbf{C} is an invertible matrix. Among the following descriptions, which is incorrect? (A) $\text{rank}(\mathbf{A})=\text{rank}(\mathbf{B})$ (B) $\text{trace}(\mathbf{A})=\text{trace}(\mathbf{B})$ (C) the eigenvalues of \mathbf{A} and \mathbf{B} are identical (D) \mathbf{A}^{-1} and \mathbf{B}^{-1} are similar if and only if \mathbf{A} is singular (E) \mathbf{A} is nonsingular if and only if \mathbf{B} is nonsingular.
6. () Which of the following is an orthogonal set of vectors? (A) $\{(1,4,2),(-2,-1,3),(6,-1,-1)\}$ (B) $\{(2,0,0),(0,3,4),(0,4,-3)\}$ (C) $\{(1,2,0,3),(-2,1,-5,-2),(4,1,0,-2)\}$ (D) $\{(8,-1,6),(-3,5,-7),(-4,9,2)\}$ (E) $\{(16,2,3,13),(5,-11,10,-8),(-9,7,6,-12)\}$.

※注意：請在答案卷上作答，寫在試題卷之答案不予採計。未依題序作答者，需註明題號。

Part II. 計算題 (每題十分)

1. Solve the differential equation $[\cos(xy) - xy \sin(xy)]dx + [-x^2 \sin(xy) + 2y]dy = 0$.
2. Solve the differential equation $y' + \frac{1}{x}y = 3x^2$.
3. Solve the differential equation $y'' + y = \delta(t - \pi) - \delta(t - 2\pi)$, $y(0) = 0$, $y'(0) = 1$.
4. Find the Fourier transform of $f(x) = \begin{cases} xe^{-x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$.
5. Find the eigenvalues and eigenvector of the matrix given by $\mathbf{A} = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 2 & -2 \\ 1 & -2 & 2 \end{bmatrix}$.
6. Use the Gram-Schmidt process to construct an orthonormal set from the following three vectors, $\mathbf{v}_1 = (0, 1, 2, -1)$, $\mathbf{v}_2 = (4, 3, 1, -1)$, $\mathbf{v}_3 = (6, 0, 3, 0)$.
7. Find the coordinate vector of $\mathbf{u} = 6x^2 - 3x + 2$ relative to the basis $\{x^2 - 1, x - 1, x^2 - x + 1\}$ in the space of 2nd-order polynomials.