

國立宜蘭大學

108 學年度研究所碩士班考試入學

工程數學(僅考矩陣與一、二階微分方程)試題

(電子工程學系碩士班)

准考證號碼：

《作答注意事項》

- 1.請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。
- 2.考試時間：100 分鐘。
- 3.本試卷共有單選題 8 題 40 分，複選題 2 題 20 分，非選擇題 4 題 40 分，共計 100 分。
- 4.請將答案寫在答案卷上。
- 5.考試中禁止使用手機或其他通信設備。
- 6.考試後，請將試題卷及答案卷一併繳交。
- 7.本試卷採雙面影印，請勿漏答。
- 8.應試時不得使用電子計算機。

I. 選單題(40%)

1. () Please identify linear of following differential Equation.

(A) $y' + 2y = 4(x^2 + 1)^2$, (B) $y'^2 = -\frac{y}{x}$, (C) $y' + \frac{x^2 - y^2}{2xy} = 0$,

(D) $(y')^2 - xy' + y = 0$, (E) $y' + \log y = x - 2$, (10%)

2. () The general solution $y' + y = 1$ is (A) $y = -1 + cx^3$, (B) $y = c(1 - x)$,
 (C) $y = e^{-x}(\cos x + \sin x)$ (D) $y = 1 - ce^{-x}$, (E) $y = x^2 - 2$,

3. () If $L^{-1}\{F(s)\} = f(t)$, the inverse Laplace transform of the given function,

$L^{-1}\left\{\frac{4s+4}{s^2+4}\right\} =$ (A) $4e^{-4t}$, (B) $4e^{-4t} - 4$, (C) $4e^{-4t}(\cos 2t + \sin 2t)$,

(D) $4\cos 2t + 2\sin 2t$, (E) $4\cosh 2t + 4\sinh 2t$, (10%)

4. () Please find the value of a such that the system $\begin{cases} x + 2y - 3z = 0 \\ 3x + y + z = 5 \\ y + az = -1 \end{cases}$ has no

solution (A) $a = -1$, (B) $a = -2$, (C) $a = 1$, (D) $a = 2$, (E) $a = 3$,

5. () Find the inner product $\vec{A} \cdot \vec{B}$ and the angle θ between two vector

$\vec{A} = 4\vec{i} + 3\vec{k}$, $\vec{B} = \vec{i} + \vec{j} - \vec{k}$

(A) $\vec{A} \cdot \vec{B} = 5\sqrt{3}$, $\theta = \cos^{-1} \frac{-1}{5\sqrt{3}}$

(B) $\vec{A} \cdot \vec{B} = 1$, $\theta = \cos^{-1} \frac{-1}{5\sqrt{3}}$

(C) $\vec{A} \cdot \vec{B} = -1$, $\theta = \cos^{-1} \frac{1}{5\sqrt{3}}$

(D) $\vec{A} \cdot \vec{B} = -1$, $\theta = \cos^{-1} \frac{-1}{5\sqrt{3}}$

(E) $\vec{A} \cdot \vec{B} = 1$, $\theta = \cos^{-1} \frac{1}{5\sqrt{3}}$

6. () Evaluation of determinants of $\begin{bmatrix} 2 & 0 & -4 & 9 \\ 4 & 5 & 1 & 0 \\ 0 & 2 & 6 & -1 \\ -3 & 8 & 9 & 1 \end{bmatrix}$

(A) 1134 (B) 1143 (C) 1153 (D) 1135 (E) 1164

7. () Solve the following D.E. $y' = y + 2y^2$ in general solution by explicit solution form.

(A) $2y = c(e^{2x} + 1)^2$, (B) $2y + y^2 = ce^x$,

(C) $y + 2y^2 = ce^x$, (D) $y^2 - x + y = c$, (E) $y^2 + \ln y = cx$,

8. () Which one of the following is a spanning set of R^3 ?

(A) $\{(1,0,-1), (0,1,1)\}$, (B) $\{(4,0,-4), (0,1,1), (2,0,-2)\}$, (C)

$\{(1,0,0), (0,1,1), (2,2,2)\}$, (D) $\{(1,3,-4), (0,1,1), (2,1,-1)\}$,

II. 複選題(20%)

- () Please identify all the even function in the following (A) e^{2x} (B) e^{-x^2}
(C) $\cosh nx$ (D) $\frac{\cosh x}{x}$ (E) $x \sinh x$ (F) $f(x) = \begin{cases} x^2, & x > 0 \\ -x^2, & x < 0 \end{cases}$
- () If $L\{f(t)\} = F(s)$, denotes the Laplace transform of the function, $f(t)$
then (A) $L\{f(at)\} = F(as)$ (B) $L\{tf(t)\} = -\frac{d}{ds}F(s)$ (C) $L\left\{\frac{d}{dt}f(t)\right\} =$
 $sF(s) + f(0)$ (D) $L\{f(t-a)\} = F(s)e^{-as}$ (E) $L\{e^{at}f(t)\} = F(s+a)$

III. 非選擇題(40%)

- Solve the O.D.E by Laplace transform $y'' + 4y' + 3y = 0$, $y(0) = 0$, $y'(0) = 4$
- Solve the differential equation : $\tilde{y}' = \tilde{A}\tilde{y} = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \tilde{y}$
- Please find the function $y(x)$ to satisfy the differential equation,
 $y'' + 5y' + 6y = 0$, $y(0) = 0$, $y'(0) = 3$. (10%)
- Please find all the eigenvalues and the corresponding eigenvectors of the matrix
 $\begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$