國　立　宜　蘭　大　學

１０３學年度研究所碩士班考試入學

工程數學試題

（電子工程學系碩士班）

准考證號碼：

《作答注意事項》

1.請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。

2.考試時間：100分鐘。

3. 本試卷共有單選題5題，一題5分，多選題6題，一題6分；計算題3題，第一題15分，第二、三題12分，共計100分。

4.請將答案寫在答案卷上。

5.考試中禁止使用大哥大或其他通信設備。

6.考試後，請將試題卷及答案卷一併繳交。

7.本試卷採雙面影印，請勿漏答。

8.應試時不得使用電子計算機。

Part I. 單選題（共25分，每題5分，答錯不倒扣）

1. $\int_0^{\infty} \cos 3t \sin 3t \, dt = ?$ (A) 0 (B) 1 (C) $\cos 3t$ (D) $\frac{1}{2}\sin 6t$ (E) $\frac{1}{9}\cos 3t \sin 3t$.

2. If a general solution is solved as $y = ce^{-x}$, the O.D.E. is (A) $y'' - y = 0$ (B) $y'' - y' = 0$, (C) $y' - y = 0$ (D) $y' + y = 0$ (E) $y'' + y' = 4x$.

3. Let $F(s) = \dfrac{3s^2 - 5s + 4}{s(s+1)(s+2)}$ be the Laplace transform of $f(t)$, then $f(\infty) = ?$ (A) 0 (B) 1 (C) 2 (D) $\frac{3}{2}$ (E) 3.

4. If $A^3 = \begin{bmatrix} 8 & 0 \\ 0 & 9 \end{bmatrix}$, then $A^{-1} = ?$ (A) $\begin{bmatrix} 9 & 0 \\ 0 & 8 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{6} & 0 \\ 0 & -\frac{1}{6} \end{bmatrix}$ (E) $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$.

5. For what value of a, is the set of vectors { (1, 2, 1), (0, 1, 1), (1, 0, a)} linear dependent ?
(A) -1 (B) 0 (C) 1 (D) -2 (E)2.

Part II. 多選題（共36分，每題有至少1個以上答案，完全正確6分，最低0分。非全對時每個選項答案選對計2分，選錯倒扣1分，未選不計分；但5個答案全選而有錯者以0分計。）

6. Which of the following differential equations is exact ? (A) $2x \ln y dx + y^{-1}x^2 dy = 0$ (B) $4y dx - x dy = 0$ (C) $xy' + y + 4 = 0$ (D) $\sin 3y dx + 3x \cos 3y dy = 0$ (E) $(1 + x^2)dy + xy^2 dx = 0$.

7. If $L\{f(t)\} \to F(s)$ denotes the Laplace transform of a given function, then (A) $L\{f(at)\} \to F(as)$ (B) $L\{tf(t)\} \to -\frac{dF(s)}{ds}$ (C) $L\{\frac{df(t)}{dt}\} \to sF(s) + f(0^+)$ (D) $L\{f(t-a)\} \to F(s)e^{as}$ (E) $L\{F(t)\} \to f(-s)$.

8. If the solution to the initial-value problem $y'' - 2y' + 10y = 0$, $y(0) = 4$, $y'(0) = 1$ is presented as $y(x) = e^{cx}(\alpha \cos ax + \beta \sin bx)$, then? (A) $a = b$ (B) $a = 2$ (C) $\alpha = 4$ (D) $\beta = -1$ (E) $c = 1$.

(翻頁仍有試題)

9. Assume A is an $m \times n$ matrix with rank r and b is a column vector. Which statements are true? (A) If $m > r$ and $n = r$, then $Ax = b$ must have no solution for some b and exactly one solution for other b. (B) If A is nonsingular, then $Ax = b$ has unique solution. (C) If n = r, then $Ax = b$ has infinite many solutions. (D) If the augmented matrix and the matrix of coefficients do not have the same rank, $Ax = b$ does not exist a solution. (E) If the augmented matrix and the matrix of coefficients have the same rank r and $r < n$, there are many solutions.

10. If $A=\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$, then (A) 2 is an eigenvalue. (B) 3 is an eigenvalue. (C) (1, -1) is an eigenvector. (D) (1, 2) is an eigenvector. (E) (1, 1) is an eigenvector..

11. Which of the following statements are true ? (A) The vectors (1, 2), (-1, 1), (3, 2) are linearly dependent in $\boldsymbol{R}^2$. (B) The vectors (1, 0, 0), (0, 2, 0), (1, 2, 0) span $\mathbf{R}^3$. (C) {(1, 0, 2), (0, 1, -3)} is a basis for the subspace of $\mathbf{R}^3$ consisting of vectors of the form (a, b, $2a$-$3b$). (D) Any set of two vectors can be used to generate a two-dimensional subspace of $\mathbf{R}^3$. (E) The rank of the matrix consists of (1, 2, 3), (0, 1, 2), (2, 5, 8) is 3.

Part III. 計算題（共39分）

1. Solve the system of differential equations. (15分)

$$\begin{cases} (D-2)x + 2Dy = 2 - 4e^{2t} \\ (2D-3)x + (3D-1) = 0 \end{cases}$$

2. Solve the following system using the method of Gauss-Jordan elimination. (12分)

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = -1 \\ x_1 + x_2 - 2x_3 = 7 \end{cases}$$

3. Find the normalized Q-R decomposition of $A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 4 & 6 \end{bmatrix}$ (12分)

※ 注意：請在答案卷上作答，寫在試題卷之答案不予採計。