基於模糊控制器之等效設計分析
解模糊化方式間的關係

陶金旺

國立宜蘭技術學院電機工程系副教授

摘要

基於模糊法則之模糊控制器的主要部分有歸屬函數、推論機制、組合運算元、與解模糊技術。這些主要部分中之每一部份，均有許多可能的選擇。在此論文中，將基於等效模糊控制器之設計，討論不同與解模糊技術間之關係。此不同與解模糊技術間關係之討論可提供設計者選擇適當解模糊技術間之參考。而等效模糊控制器之設計亦提供更彈性之模糊控制器設計方式。

關鍵詞：模糊控制器、解模糊技術、歸屬函數

Relationship Analyses Of Defuzzification Methods Based On The Equivalent Designs of Fuzzy Controllers

C.W. Tao

Associate Professor, Department of Electrical Engineering, National Ilan Institute of Technology

Abstract

The major components of fuzzy controllers based on if-then rules are membership functions for fuzzification, inference mechanisms, composition operators, and defuzzification techniques. Each of these major parts may be selected from several possible candidates. In this paper, we will focus on the relationship analyses of the different defuzzification methods based on the equivalent design of fuzzy controllers. The relationship analyses of the different defuzzification methods give the idea for designers to select the appropriate defuzzification strategy. And the design approaches of equivalent controllers provide more flexibility to construct fuzzy controllers.

Key Words : Fuzzy Controller, Defuzzification Technique, Membership Function

I. Introduction

Fuzzy controllers have been successfully utilized in the complex or ill-defined dynamic systems for the last several years [1,4,6,8,9]. The major parts of fuzzy controllers based on fuzzy if-then rules are membership functions for fuzzification, inference mechanisms, composition operators, and defuzzification techniques. Each of these major parts may be selected from several possible candidates. One of key issues for the effective design of fuzzy controllers is to determine the suitable combination of the alternatives for these major components. Since there is in general no systematic approach to obtain an effective design for fuzzy controllers, it is an interesting research topic to find out the relationships between possible candidates of each major component of fuzzy controllers. In the book of Driankov and his colleague [2], the different defuzzification techniques are evaluated with respect to some performance criteria. Yager and Filev [10] propose the basic defuzzification distribution (BADD) transformation to generalize the defuzzification approach. However, these analyses for the defuzzification methods are only componentwise. These componentwise analyses can not give the clear idea for the designer to select the appropriate defuzzifier. In this paper, the relationships between the different defuzzification methods are analyzed based on the equivalent design of fuzzy controllers. In our study, defuzzifier B is said to be more flexible than the defuzzifier C if there exists an equivalent fuzzy controller with the defuzzifier B for every fuzzy controller with the defuzzifier C. That is, we try to find out the systemwise relationships between different defuzzification techniques. Even though the systemwise relationships between the different defuzzifiers are found only when some conditions are satisfied, the analyses would suggest clearly the proper defuzzification method for the system designer since the conditions are not very restrictive. Moreover, the design approaches of equivalent controllers provide more flexibility to construct fuzzy controllers.

The remainder of this paper is organized as follows. The relationships between defuzzification techniques are defined in Section II. Section III presents the relationship between the expected value and centroid defuzzifiers. The relationship between the height and centroid defuzzifiers are provided in Section IV. Section V discusses the relationship between the mean of maximum [7] and centroid defuzzification techniques. Section VI states a conclusion.

II. Definitions of Relationships

Based on the definition of the equivalent fuzzy controllers,

Definition 1 Two fuzzy controller, $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$,are defined to be equivalent (denoted by $\boldsymbol{F_1} \Leftrightarrow \boldsymbol{F_2}$) if the input-output mappings of $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are the same.

the relationships between the defuzzification techniques can be defined as follows:

Definition 2 For two defuzzification methods $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$, $\boldsymbol{D}_2$ is defined to be more flexible than $\boldsymbol{D}_1$ (denoted by $\boldsymbol{D}_1 \Rightarrow \boldsymbol{D}_2$) if for every fuzzy controller designed with $\boldsymbol{D}_1$, there exists an equivalent fuzzy controller designed with $\boldsymbol{D}_2$.

Definition 3 For two defuzzification methods $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$, $\boldsymbol{D}_2$ is defined to be conditionally more flexible than $\boldsymbol{D}_1$ (denoted by $\boldsymbol{D}_1 \to \boldsymbol{D}_2$) if $\boldsymbol{D}_1 \Rightarrow \boldsymbol{D}_2$ is true only under certain conditions.

Definition 4 For two defuzzification methods $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$, if $\boldsymbol{D}_1 \to \boldsymbol{D}_2$ and $\boldsymbol{D}_2 \to \boldsymbol{D}_1$ are true then $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are defined to be conditionally equivalent (denoted by $\boldsymbol{D}_1 \leftrightarrow \boldsymbol{D}_2$) .

With these definitions, the relationships between the centroid defuzzification ($\boldsymbol{CT}$) method and other defuzzification methods (expected value ($\boldsymbol{EV}$), height ($\boldsymbol{HT}$), and mean of maximum ($\boldsymbol{MM}$)) are discussed in the next three sections.

III. Relationship between *EV* and *CT*

In this section, the fuzzy controller ***EVFC*** using the expected value method (***EV***) as the defuzzification technique and the fuzzy controller ***CTFC*** with the centroid defuzzification method ***CT*** are first defined. Then the relationship between ***EV*** and ***CT*** is presented. To have the fuzzy controllers ***EVFC*** and ***CTFC*** defined, the universes of discourse of input variables x_j, j = 1,2,...,p, are fuzzily partitioned into n fuzzy sets, A_j^β, β =1,2,...,n. Also, the operators ***min*** and ***max*** are adopted as the inference and composition operators, respectively.

1. Definition of *EVFC*

For the fuzzy controller ***EVFC***, the output variable y is fuzzily partitioned into n fuzzy sets B^β, with membership functions $\mu_{B^\beta}(y)$, β =1,2,... n. Thus, the ***EVFC*** is defined to be a fuzzy controller based on a fuzzy rule base with the i^{th} fuzzy rule as

$$R_i: \text{If } x_1 \text{ is } \Lambda_{1i}, ..., x_p \text{ is } \Lambda_{pi}, \text{ then } y \text{ is } \Gamma_i,$$

where

$$\Lambda_{ji} \in \{ A_j^1, A_j^2, ..., A_j^n \}$$

and

$$\Gamma_i \in \{ B^1, B^2, ..., B^n \}$$

And the membership function for the control action is

$$\mu_{y_{out}}(y) = \max_i (\min(\mu_{\Lambda_{1i}}(x_1), ..., \mu_{\Lambda_{pi}}(x_p), \mu_{\Gamma_i}(y))) \quad (1)$$

where the index i indicates the i^{th} rules. If the universe of discourse of output variable y is discrete, then the control action y_{out} of the ***EVFC*** with the expected value defuzzification technique is

$$y_{out} = \frac{\sum_l f(y_1)\mu_{yout}(y_l)}{\sum_l \mu_{yout}(y_l)} \quad (2)$$

where f is a function of y.

2. Definition of *CTFC*

The Let the output variable z of the fuzzy controller ***CTFC*** be fuzzily partitioned into fuzzy sets D^β with membership functions $\mu_{D^\beta}(z)$, β =1,2,...,n. Then the i^{th} fuzzy if-then rules for the ***CTFC*** become

$$R_i: \text{If } x_1 \text{ is } \Lambda_{1i}, ..., x_p \text{ is } \Lambda_{pi}, \text{ then } z \text{ is } \Delta_i$$

where Δ_i is the corresponding output fuzzy set and

$$\Delta_i \in \{D^1, D^2, ..., D^n\}$$

With the membership function of the control action

$$\mu_{z_{out}}(z) = \max_i(\min(\mu_{\Lambda_{1i}}(x_1), ..., \mu_{\Lambda_{pi}}(x_p), \mu_{\Delta_i}(z))), \quad (3)$$

the crisp control action of ***CTFC*** using the centroid defuzzification technique is

$$z_{out} = \frac{\sum_l z_l \mu_{z_{out}}(z_l)}{\sum_l \mu_{z_{out}}(z_l)}. \tag{4}$$

3. Relationship between CT and EV

From the definitions of the fuzzy controllers ***EVFC*** and ***CTFC*** in the subsections **Ⅲ.1** and **Ⅲ.2**, the relationship of the defuzzification methods, ***CT*** and ***EV*** can be stated in the Theorem 1. And the proof of the Theorem 1 is also provided.

Theorem 1 *The defuzzification methods **CT** and **EV** are equivalent if the condition*

- *the function* f *in the Eq. 2 is one-to-one and onto*

*is satisfied. That is, **CT** and **EV** are conditionally equivalent.*

Proof：

(***EV*** $\longrightarrow$ ***CT***)：

Let $z = f(y)$ and the membership functions μ_{D^b} ,

$$\mu_{D^b}(z) = \mu_{B^b}(f^{-1}(z)) = \mu_{B^b}(y)$$

since f is one-to-one and onto, the inverse function f^{-1} exists and there is one and only one corresponding y for every z. Also, the fuzzy set Δ_i in the rule i of ***CTFC*** is defined to have

$$\mu_{\Delta_i}(z) = \mu_{\Gamma_i}(y), \quad \forall z = f(y),$$

where Γ_i is the output fuzzy set used in the rule i of ***EVFC***. Thus, for every input vector $X = (x_1, x_2, ..., x_n),$, the membership function of the control action of ***CTFC***,

$$\begin{aligned} \mu_{z_{out}}(z) &= \max_i (\min(\mu_{\Lambda_{1i}}(x_1), ..., \mu_{\Lambda_{pi}}(x_p), \mu_{\Delta_i}(z))) \\ &= \mu_{y_{out}}(y) \end{aligned} \tag{5}$$

and the crisp control action of ***CTFC*** using the centroid defuzzification technique is

$$\begin{aligned} z_{out} &= \frac{\sum_l z_l \mu_{z_{out}}(z_l)}{\sum_l \mu_{z_{out}}(z_l)} \\ &= \frac{\sum_l f(y) \mu_{y_{out}}(y_l)}{\sum_l \mu_{z_{out}}(y_l)} = y_{out} \end{aligned} \tag{6}$$

The Eq. 6 indicates that for every fuzzy controller ***EVFC*** designed with ***EV***, there exists an equivalent fuzzy controller designed with ***CT*** if ***f*** is one-to-one and onto. Therefore, ***EV*** $\longrightarrow$ ***CT*** is proved.

(***CT*** $\longrightarrow$ ***EV***)：

With the similar approach, ***CT*** $\rightarrow$ ***EV*** can be easily proved if ***f*** is one-to-one and onto.

Moreover, the discussion in this subsection can be applied to the continuous type fuzzy controllers ***EVFC*** and ***CTFC*** with inference and composition operators other than the ***min*** and ***max*** operators used here. That is, the restriction on the type of inference and composition operators is not necessary for the discussion here.

IV. Relationship between *HT* and *CT*

With the inference and composition operators chosen to be the ***min*** and ***max*** operators respectively, the relationship between the height (***HT***) and centroid (***CT***) defuzzification techniques are introduced in this section. For the construction of the fuzzy controller ***HTFC*** with ***HT***, the universes of discourse of input variables $x_j,\ j = 1,2,\ldots,p,$ are fuzzily partitioned into n fuzzy sets, $A_j^{\beta}\ ,\beta = 1,2,\ldots,n.$

1. Definition of *HTFC*

The fuzzy controller ***HTFC*** is defined to be a fuzzy controller with the height defuzzification technique[2]. If the output variable y of ***HTFC*** is fuzzily partitioned into n fuzzy sets B^{β}, with membership functions $\mu_{B^{\beta}}(y),\ \beta = 1,2,\ldots,n,$ then the fuzzy rule base ***HTFC*** contains fuzzy rules with the form

$$R_i : \text{If } x_1 \text{ is } \Lambda_{1i},\ldots,x_p \text{ is } \Lambda_{pi},\ \text{then } y \text{ is } \Gamma_i,$$

where

$$\Lambda_{ji} \in \left\{A_j^1, A_j^2, \ldots, A_j^n\right\}$$

and

$$\Gamma_i \in \left\{B^1, B^2, \ldots, B^n\right\}$$

And the degree of match of the i^{th} rule antecedent is

$$f_i(X) = \min(\mu_{\Lambda_{1i}}(x_1),\ldots,\mu_{\Lambda_{pi}}(x_p)),\ X = (x_1, x_2, \ldots, x_p). \tag{7}$$

With the height defuzzification technique utilized , the control y_{out} of ***HTFC*** is

$$y_{out} = \frac{\sum_i y_i f_i(X)}{\sum_i f_i(X)} \tag{8}$$

where y_i is the point at which the $\mu_{\Gamma_i}(y_i)$ reaches the maximum value. Note that if there is an interval of y, $[y_l\, y_u]$, such that $\mu_{\Gamma_i}(y)$ reaches the maximum value for every $y \in [y_l\, y_u]$, then y_i is defined to be the middle point of $[y_l\, y_u]$

2. Relationship between *HT* and *CT*

Let the fuzzy controller ***CTFC*** be defined as in the subsection Ⅲ.2.Then the following Theorems point out the relationship between ***CT*** and ***HT***.

Theorem 2 *The defuzzification method* ***CT*** *is more flexible than* ***HT***(***HT*** $\Rightarrow$ ***CT***).

Proof :

Let the output fuzzy set Δ_i correspond with the output fuzzy set $\Gamma_i,\ \forall i$. And the membership function of the support of the output fuzzy set Δ_i in the fuzzy rule i is symmetrical to the point z_i, and $\mu_{\Lambda_{1i}}(z_i)$ reaches the maximum value. Then the control action of the fuzzy controller ***CTFC*** is

$$z_{out} = \frac{\sum_i z_i \min(\mu_{\Lambda_{pi}}(x_p))}{\sum_i \min(\mu_{\Lambda_{1i}}(x_1),\ldots,\mu_{\Lambda_{pi}}(x_p))}$$

$$= \frac{\sum_i z_i f_i(X)}{\sum_i f_i(X)} \tag{9}$$

as mentioned in Kosko's work[5].Since $\mathbf{D}_i$ is the corresponding fuzzy set of $\mathbf{G}_i$

$$z_i = y_i, \forall_i$$

where yi is defined as in subsection 4.1. Therefore,

$$z_{out} = \frac{\sum_i z_i f_i(X)}{\sum_i f_i(X)}$$

$$= \frac{\sum_i y_i f_i(X)}{\sum_i f_i(X)} = y_{out} \tag{10}$$

And $HT \Rightarrow CT$ is proved.

Note that Theorem 2 is still correct if other inference and composition operators are utilized.

Theorem 3 *If the condition*

- *the membership functions of the supports of the output fuzzy sets are symmetrical to the point at which the membership values reach the maximum values*

is satisfied, the defuzzification method HT *is conditionally more flexible than* $CT(CT \rightarrow HT)$.

It will be straightforward to show that $CT \rightarrow EV$, and the proof is omitted for simplicity. From the Theorem 2 and 3, Theorem 4 can be obtained easily.

Theorem 4 *If the condition*

- *the membership functions of the supports of the output fuzzy sets are symmetrical to the point at which the membership values reach the maximum values*

is satisfied, the defuzzification methods CT and HT are conditionally equivalent $(CT \leftrightarrow HT)$.

V. Relationship between *HT* and *MM*

The definitions of the fuzzy controller $(MMFC)$ with the mean of maximum defuzzifier and the fuzzy controller $(CTFC_2)$ with centroid defuzzification method are presented in subsections V.1 and V.2, respectively. Then the relationship between CT and MM is proposed. And a simple example to illustrate the relationship between CT and MM is provided to end this section.

1. Definition of *MMFC*

Let the universes of discourse of input variable $x_j, j = 1,2,..., p$ and output variable y be fuzzily partitioned into n fuzzy sets, A_j^b and B^b, $b = 1,2,..., n,$ respectively. And the fuzzy rule base of $MMFC$ consists of n_c fuzzy if-then rules with the form

$$R_i : \text{If } x_1 \text{ is } \mathbf{L}_{1i},..., \ x_p \text{ is } \mathbf{L}_{pi}, \text{ then } y \text{ is } \mathbf{G}_i,$$

where

$$\mathbf{L}_{ji} \in \{A_j^1, A_j^2, ..., A_j^n\}$$

and

$$\mathrm{G}_i \in \{B^1, B^2, \ldots, B^n\}.$$

Suppose that ***min*** and ***max*** are chosen to be the inference operator and the composition operator respectively. Then the membership function for the control action is

$$m_{yout}(y) = \max_i(\min(m_{\mathrm{L}_{1i}}(x_1), \ldots, m_{\mathrm{L}_{pi}}(x_p), m_{\mathrm{G}_i}(y))) \qquad \textbf{(11)}$$

where the index i indicates the i^{th} rule.If the universe of discourse of output variable y is discrete and the mean of maximum defuzzification technique [7] is utilized, then the control action y_{out} is

$$y_{out} = \sum_{t=1}^{m} \frac{y_l}{m} \qquad \textbf{(12)}$$

where y_l is the point at which $m_{y_{out}}(y_l)$ reaches the maximum value, and m is the number of such points.

2. Definition of $CTFC_2$

With the same partition in the subsection Ⅴ.1, the universes of discourse of the input variable x_j, $j = 1,2,\ldots, p,$ of the fuzzy controller $CTFC_2$ are fuzzily partitioned into n fuzzy sets, A_j^b, $b = 1,2,\ldots,n$. And the universe of the output variable z is partitioned into n_c fuzzy sets, D_i, $i = 1,2,\ldots,n_c$. That is, there is one specific output fuzzy set for each fuzzy if-then rule. Thus, the fuzzy rule base for the fuzzy controller $CTFC_2$ consists of n_c rules, and each of the fuzzy if-then rules has the form

R_i : If x_1 is $\mathrm{L}_{1i}, \ldots,$ x_p is L_{pi}, then z is D_i,

where

$$A_{ji} \in \{A_j^1, A_j^2, \ldots, A_j^n\}$$

If the fuzzy region

x_1 is $\mathrm{L}_{1i}, \ldots,$ x_p is L_{pi},

is defined to be a new fuzzy set Q_i, the fuzzy rules become to have the new form as

R_i : If X is Q_i, then z is D_i,

where

Q_i represents x_1 is $\mathrm{L}_{1i}, \ldots,$ x_p is L_{pi}.

With the membership function of the control action

$$m_{z_{out}}(z) = \max_i(\min(m_{\mathrm{Q}_i}(X), m_{\mathrm{D}_i}(z))), \qquad (13)$$

the crisp control action of $CTFC_2$ using the centroid defuzzification technique is

$$z_{out} = \frac{\sum_l z_l m_{z_{out}}(z_l)}{\sum_l m_{z_{out}}(z_l)} \qquad (14)$$

3. Relationship between CT and MM

Theorem 5 the CT and MM with the fuzzy controller $CTFC_2$ and $MMFC$ defined above.

Theorem 5 *The defuzzification methods* CT *is conditionally more flexible than the defuzzifier* MM $(MM \rightarrow CT)$, *if the conditions*

- *the universes of the output variables are discrete*
- *the inference operator is min and the composition operator is max are satisfied.*

Proof:

If the fuzzy controller $MMFC$ is constructed as in the subsection Ⅵ.1, we can design a fuzzy controller $CTFC_2$ defined in subsection Ⅵ.2 with

- the membership function $m_{\Theta_i}(X)$ of the fuzzy set $\mathbf{Q}_i$,

$$m_{\Theta_i}(X) = \begin{cases} 1, & \text{if} \quad m_{\Theta_i}(X) = \max_i (\min(m_{\Lambda_{1i}}(x_1), ..., m_{\Lambda_{pi}}((x_p)) \\ & 0, \quad \text{otherwise} \end{cases}$$

- z and y have the same discrete universe
- the membership function for the fuzzy set $\mathbf{D}_i(z)$ of the output variable z is specified to be

$$m_{D_i}(z) = \begin{cases} 1, \text{if} & m_{G_i}(z) \geq \max_i (\min(m_{L_{1i}}(x_1), ..., m_{L_{pi}}(x_p)) \\ & 0, \text{otherwise} \end{cases} \tag{15}$$

Then the membership functions for the fuzzy sets of the control action z_{out} is defined to be

$$\begin{aligned} & m_{zout}(z) \\ &= \max(\min(m_{Q_i}(X), m_{D_i}(z))) \\ &= \; = \begin{cases} 1, \text{if} & m_{Q_i}(x) = 1 \quad \text{and} \quad m_{Di} = 1 \\ & 0, \text{otherwise} \end{cases} \end{aligned} \tag{16}$$

and the control action of $CTFC_2$ becomes

$$\begin{aligned} Z_{out} &= \frac{\sum_k z_k m_{z_{out}}(z_k)}{\sum_k m_{z_{out}}(z_k)} \\ &= \frac{\sum_l z_l}{\sum_l 1} \end{aligned} \tag{17}$$

where z_l is the value such that $m_{z_{out}}(z_l) = 1$. From Eq. 15, it can be seen that

$$z_1 = y_1, \; \forall l; \text{ and}$$
$$\sum_l 1 = m,$$

that is, $z_{out} = y_{out}$. Thus, the fuzzy controllers $CTFC_2$ and $MMFC$ have the same input-output mappings. And $MM \rightarrow CT$ is proved.

To illustrate the relationship between the defuzzification methods, MM and CT, a simple example is provided.

Example 1

In this example, the fuzzy controller $MMFC$ *is assumed for simplicity to be a single-input/single-output fuzzy controller with the membership functions for the fuzzy sets of the input variables* x *and the output variable* y *shown in Figure 1. Let the universes of discourse of input variable* x *and output variable* y *be fuzzily partitioned into seven fuzzy sets which are ``negative big (nb)'', ``negative medium (nm)'', ``negative small (ns)'', ``zero (ze)'', ``positive'small (ps)'', ``positive medium (pm)'', and ``positive big (pb)''. And, the fuzzy if-then rules for* $MMFC$ *have the form.*

$$R_i: \text{If } x \text{ is } \mathbf{L}_i, \text{ then } y \text{ is } \mathbf{G}_i \tag{18}$$

where $\mathbf{L}_i$ *and* $\mathbf{G}_i$ *represent one of the fuzzy sets of the input variable and output variable, respectively. Then the membership function for the control action is*

$$m_{y_{out}}(y) = \max_i(\min(m_{L_i}(x), m_{G_i}(y))) \quad (19)$$

where the index i indicates the i^{th} *rule. Since the universe of discourse of output variable* y *is discrete, the control action* y_{out} *of* $MMFC$ *is*

$$y_{out} = \sum_{t=1}^{m} \frac{y_l}{m} \quad (20)$$

where y_1 *is the point at which* $m_{y_{out}}(y_l)$ *reaches the maximumvalue, and* m *is the number of such points. For the fuzzy controller* $MMFC$ *in this example, we can construct the fuzzy controller* $CTFC_2$ *with*

- *the membership function* $m_{Q_i}(x)$ *of the fuzzy set* Q_i *(see Figure 3)* as

$$m_{Q_i}(x) = \begin{cases} 1, & \textbf{\textit{if}} \quad m_{Q_i}(x) = \max_i\left(\min\left(m_{L_i}(x)\right)\right) \\ 0, & \textbf{\textit{otherwise}} \end{cases}$$

- *the membership function* $m_{D_i}(z)$ *for the fuzzy set* D_i *of the output variable z is specified to be*

$$m_{\Theta_i}(x) = \begin{cases} 1, & \textbf{\textit{if}} \quad m_{\Gamma_i}(z) \geq \max_i(\min(m_{\Lambda_i}(x))) \\ 0, & \textbf{\textit{otherwise}} \end{cases} \quad (21)$$

Then the membership functions for the fuzzy sets of the control action z_{out} *can be obtained as*

$$\begin{aligned} & m_{z_{out}}(z) \\ &= \max_i(\min(m_{\Theta i}(x), m_{\Delta i}(z))) \\ &= \begin{cases} 1, & \textbf{\textit{if}} \quad m_{\Theta_i}(x) = 1 \quad \textbf{\textit{and}} \quad m_{\Delta_i}(z) = 1 \\ 0, & \textbf{\textit{otherwise}} \end{cases} \end{aligned} \quad (22)$$

And the control action of $CTFC_2$ is

$$z_{out} = \frac{\sum_k z_k m_{z_{out}}(z_k)}{\sum_k m_{z_{out}}(z_k)} = \frac{\sum_l z_l}{\sum_l 1} \quad (23)$$

where z_l *is the value such that* $m_{z_{out}}(z_l) = 1$. *To compare the* $MMFC$ *and* $CTFC_2$ *,we can find that*

1. *for every input* x_1, *there are two fuzzy rules are activated. Let* c_i *be the point at which the membership values of the two activated input fuzzy sets are equal (see Figure 2). Then the control action* $y_{out} = d_i$ *in Eq. 20 is a constant for every* $x_1 > c_i$. *The* $y_{out} = d_{i-1}$ *for every* $x_1 < c_i$ *is also a constant.*
2. *rom Figure 3, the control action* $z_{out} = d_i$ *is a constant for every* $x_1 \rangle c_i$, *and* $z_{out} = d_{i-1}$ *for every* $x_1 \langle c_1$ *is also a constant.*
3. *when* $x_1 = c_1, y_{out}$ *is equal to* $(d_i + d_{i-1})/2$
4. *Again from Figure 3 and Eq.17,the control action* $z_{out} = (d_i + d_{i-1})/2$.

Thus, the $MMFC$ and $CTFC_2$ have same input-output mapping.

VI. Conclusions

A methodology for the analyses of the relationships between different defuzzification is proposed based on the equivalent design of fuzzy controllers. The relationships between the centroid defuzzifier and other defuzzification techniques (expected value, height, and mean of maximum) are presented to give the idea for designers to select the appropriate defuzzification strategy. And the equivalent design of fuzzy controllers provide more freedom for the construction of fuzzy controllers.

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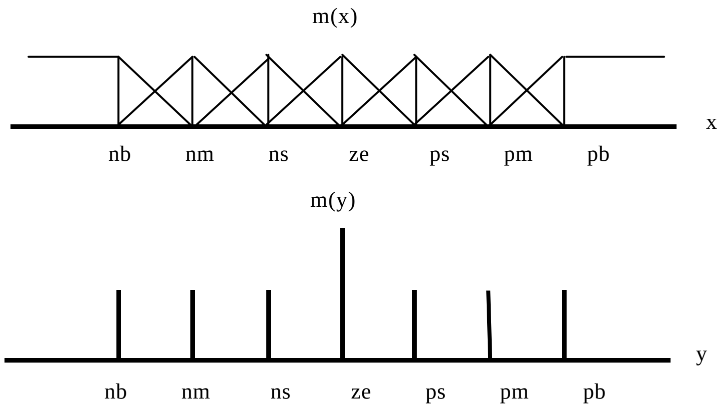

Fig 1 Membership functions of the fuzzy controller ***MMFC***.

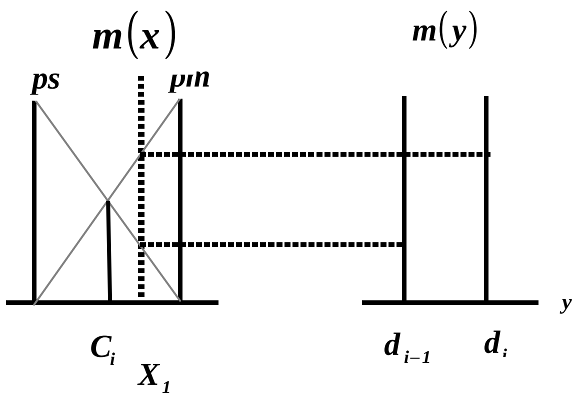

Fig 2 Graphical representation of fuzzy reasoning.

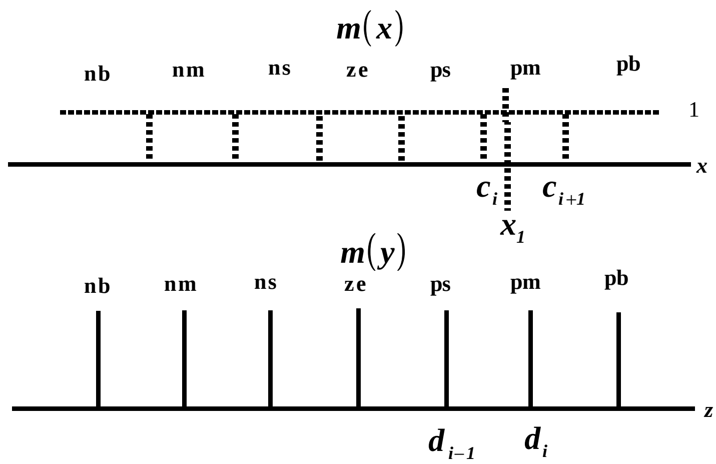

Fig 3 Membership functions of the fuzzy controller ***CTFC*$_2$**.