

1. Single Choice Problems (each sub-problem: 5 points)

(1) Let $n \in \mathbb{N}$, $x \in \mathfrak{R}$ and $y \in \mathfrak{R}$. Then the $(r+1)^{\text{th}}$ item of polynomial $(x+y)^n$ (arranged in the descending order of the powers of x) can be expressed as?

(A) $\binom{n}{r} x^{n-r-1} y^{r+1}$ (B) $\binom{n}{r+1} x^{n-r} y^r$ (C) $\binom{n}{r+1} x^{n-r-1} y^{r+1}$ (D) $\binom{n}{r} x^{n-r} y^r$

(2) Let U, V be arbitrary sets, then based on the *De Morgan* laws we have $(U \cap V)' = ?$

(A) $U' \cup V'$ (B) $U' \cap V'$ (C) $U \cup V'$ (D) $U' \cap V$

(3) Let n be an integer represented in base 10 as a sequence of t decimal digitals, i.e., $d_1 d_2 d_3 \dots d_t$. Then $n \bmod 9$ can be expressed as?

(A) $\left(\sum_{i=1}^t d_i\right) \bmod 9$ (B) $\left(\sum_{i=1}^t d_i\right) \bmod 3$ (C) $\left(\prod_{i=1}^t d_i\right) \bmod 9$ (D) $\left(\prod_{i=1}^t d_i\right) \bmod 3$

(4) Assume that n is an exact power of 2, which is the solution of the recurrence relation shown below?

$$\Gamma(n) = \begin{cases} 2 & \text{if } n = 2 \\ 2\Gamma(n/2) + n & \text{if } n = 2^k, \forall k > 1 \end{cases}$$

(A) $\Gamma(n) = 2 \lg n$ (B) $\Gamma(n) = n \lg n$ (C) $\Gamma(n) = n \lg \lg n$ (D) $\Gamma(n) = n \lg \lg \lg n$

(5) When use a binary search algorithm to find an element in an N -element list, how many elements in the list will be examined before the algorithm returns a failure in finding the element?

(A) N (B) $\log N$ (C) $N/2$ (D) $\log(N/2)$

(6) Let $k \in \mathbb{N}$, $n \in \mathbb{N}$ and $r \in \mathfrak{R}$. Then which summation formula below is wrong?

(A) $\sum_{k=0}^n ar^k = \frac{ar^{n+1} - a}{r - 1}, r > 1$

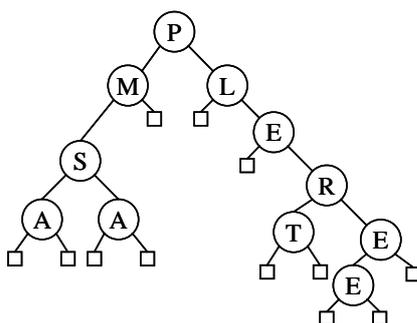
(B) $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$

(C) $\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$

(D) None

2. The **edge connectivity** of an undirected graph denotes the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1. Accordingly, the edge connectivity of a cyclic chain of vertices is _____. (7 points)

3. Visiting the tree below in postorder gives the result A _____ P (18 points)



4. If we flip a coin, there is probability p that it comes up heads and probability q that it comes up tails, where $p + q = 1.0$; i.e., this process have just two outcomes. If we toss the coin n times and assume that different coin tosses are always independent. Then the chance of obtaining exactly k tails in n tosses is _____. (15 points)

5. The 8-bit two's complement notation of $(7)_{10}$ is _____. (10 points)

6. The harmonic numbers $H_j, j = 1, 2, 3, \dots$ are defined as follows:

$$H_j = 1 + 1/2 + 1/3 + \dots + 1/j.$$

Show that $H_{2^n} \geq 1 + \frac{n}{2}$, where n is a nonnegative integer. (10 points)

7. Given an undirected graph $G = (V, E)$ with $|V|$ nodes and $|E|$ edges. To make sure that G is connected, the value of $|E|$ is at least _____. (10 points)