解下列微分方程式

1. $\left(e^{x+y} - y\right)dx + \left(xe^{x+y} + 1\right)dy = 0$ (15%)

2. $y' + y/x^2 = 2xe^{1/x}$ (10%)

3. $\left(D^2 - 2D + I\right)y = x^2 + x^{-2}e^x$ (15%)

4. $\begin{aligned} y'_1 &= y_1 + 2y_2 + e^{2t} \\ y'_2 &= -y_2 + 1.5e^{-2x} \end{aligned}$ (15%)

5. $x^2 y'' + 6xy' + \left(4x^2 + 6\right)y = 0$ (Try Frobenius method) (15%)

6. $x\dfrac{\partial w}{\partial x} + \dfrac{\partial w}{\partial t} = xt, \quad w(x,0) = 0 \ if \ x \geq 0, \quad w(x,t) = 0 \ \ if \ t \geq 0$ (15%)

7. $y'' + 9y = 8\mathrm{Sin}t \quad if \ 0 < t < \pi \ and \ = 0 \ if \ t > \pi; \quad y(0) = 0, \ y'(0) = 4$

(15%)