1. Single Choice Problems (each sub-problem: 5 points)

(1) Let $n \in \aleph$, $x \in \Re$ and $y \in \Re$. Then the $(r+1)^{\text{th}}$ item of polynomial $(x+y)^n$ (arranged in the descending order of the powers of x) can be expressed as?

(A) $\binom{n}{r} x^{n-r-1} y^{r+1}$ (B) $\binom{n}{r+1} x^{n-r} y^{r}$ (C) $\binom{n}{r+1} x^{n-r-1} y^{r+1}$ (D) $\binom{n}{r} x^{n-r} y^{r}$

(2) Let U, V be arbitrary sets, then based on the *De Morgan* laws we have $(U \cap V)' =$?

(A) $U' \cup V'$ (B) $U' \cap V'$ (C) $U \cup V'$ (D) $U' \cap V$

(3) Let n be an integer represented in base 10 as a sequence of t decimal digitals, i.e., $d_1\ d_2\ d_3 \ldots d_t$. Then n mod 9 can be expressed as?

(A) $\left(\sum_{i=1}^{t} d_i\right) \bmod 9$ (B) $\left(\sum_{i=1}^{t} d_i\right) \bmod 3$ (C) $\left(\prod_{i=1}^{t} d_i\right) \bmod 9$ (D) $\left(\prod_{i=1}^{t} d_i\right) \bmod 3$

(4) Assume that n is an exact power of 2, which is the solution of the recurrence relation shown below?

$$\Gamma(n) = \begin{cases} 2 & \text{if } n = 2 \\ 2\Gamma(n/2) + n & \text{if } n = 2^k, \forall k > 1 \end{cases}$$

(A) $\Gamma(n) = 2\lg n$ (B) $\Gamma(n) = n\lg n$ (C) $\Gamma(n) = n\lg\lg n$ (D) $\Gamma(n) = n\lg\lg\lg n$

(5) When use a binary search algorithm to find an element in an N-element list, how many elements in the list will be examined before the algorithm returns a failure in finding the element?

(A) N (B) $\log N$ (C) $N/2$ (D) $\log(N/2)$

(6) Let $k \in \aleph$, $n \in \aleph$ and $r \in \Re$. Then which summation formula below is wrong?

(A) $\sum_{k=0}^{n} ar^k = \frac{ar^{n+1} - a}{r-1}, r > 1$

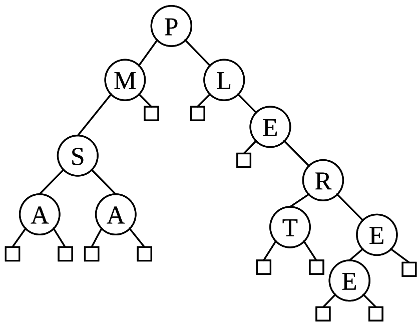
(B) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

(C) $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

(D) None

2. The **edge connectivity** of an undirected graph denotes the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1. Accordingly, the edge connectivity of a cyclic chain of vertices is ___. (7 points)

3. Visiting the tree below in postorder gives the result A _ _ _ _ _ _ _ _ _ _ P
(18 points)



4. If we flip a coin, there is probability p that it comes up heads and probability q that it comes up tails, where $p + q = 1.0$; i.e., this process have just two outcomes. If we toss the coin n times and assume that different coin tosses are always independent. Then the chance of obtaining exactly k tails in n tosses is ______. (15 points)

5. The 8-bit two's complement notation of $(7)_{10}$ is _ _ _ _ _ _ _ _. (10 points)

6. The harmonic numbers H_j, $j = 1, 2, 3, \ldots$ are defined as follows:
$H_j = 1 + 1/2 + 1/3 + \ldots + 1/j$.
Show that $H_{2^n} \geq 1 + \frac{n}{2}$, where n is a nonnegative integer. (10 points)

7. Given an undirected graph $G = (V, E)$ with $|V|$ nodes and $|E|$ edges. To make sure that G is connected, the value of $|E|$ is at least ______. (10 points)