1. (15%) A system may or may not be

   - Memoryless
   - Time invariant
   - Linear
   - Causal
   - Stable

   Determine which of these properties hold and which do not hold for the following continuous-time systems. Justify your answers. $y(t)$ denotes the system output and $x(t)$ is the system input.

   $$y(t) = x(t-2) + x(2-t)$$

2. (12%) Compute the convolution of the following two signals:

   $$x(t) = \begin{cases} 1 & 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$

   $$h(t) = \begin{cases} 1 & 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$

3. (12%) Consider a causal LTI system whose input $x[n]$ and output $y[n]$ are related by the difference equation

   $$y[n] = \frac{1}{3}y[n-1] + x[n]$$

   Determine $y[n]$, if $x[n] = \delta[n-1]$.

4. (14%) A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are

   $$a_0 = 1,\ a_2 = a_{-2} = -1,\ a_3 = a^*_{-3} = e^{j\frac{\pi}{3}}$$

   (a) Express $x(t)$ in the form

   $$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k)$$

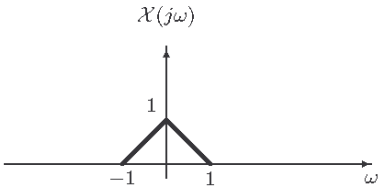
   where $\omega_k = \frac{2k\pi}{T}$

   (b) Determine the Fourier transform of $x(t)$?

5. (12%) Determine the Fourier transform $X(j\omega)$ of the unit step $x(t) = u(t)$.

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

6. (20%) Supppose that $X(j\omega)$ (the Fourier transform of $x(t)$) is depicted in the following figure. $y(t) = x(t)p(t)$. Sketch the spectrum of $y(t)$ for each of the following choice of $p(t)$



(a) $p(t) = \cos(5t)$

(b) $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2\pi k)$

7. (15%) Consider a discrete causal LTI system whose input $x[n]$ and output $y[n]$ are related by the following difference equation

$$y[n] = x[n] + x[n-1] + x[n-2]$$

(a) What is the impulse response of the above LTI system? (5%)

(b) Determine the frequency response of the above LTI system, and show that it is a low-pass filter. (10%)