# 國　立　宜　蘭　大　學

# 107 學年度暑假轉學招生考試

（考生填寫）
准考證號碼：

## 微　積　分　試　題

## 《作答注意事項》

1.請先檢查准考證號碼、座位號碼及答案卷號碼是否相符。

2.考試時間：80 分鐘。

3.本試卷共有 20 題選擇題，一題 5 分，共計 100 分。

4.請將答案寫在答案卷上（於本試題上作答者，不予計分）。

5.考試中禁止使用手機或其他通信設備。

6.考試後，請將試題卷及答案卷一併繳交。

7.本試卷採雙面影印，請勿漏答。

8.應試時不得使用電子計算機。

1. Find the absolute maximum value of the function: $f(x) = \sqrt{2}\ x + \cos 2x$, $x \in [0, \frac{\pi}{2}]$. (A) $\frac{\sqrt{2}}{2}\pi - 1$ (B) $\frac{\sqrt{2}(3\pi-4)}{8}$ (C) $\frac{\sqrt{2}(3\pi+4)}{8}$ (D) $\frac{\sqrt{2}(\pi+4)}{8}$ (E) $\frac{\sqrt{2}(\pi-4)}{8}$.

2. $\lim_{x\to\infty}(3x+2)^{\frac{1}{\ln x}}$ = ? (A) 0 (B) $e^{-1}$ (C) 1 (D) $e$ (E) $e^{\frac{1}{2}}$

3. $\lim_{x\to\infty} x^2 - x\sqrt{x^2-2}$ = ? (A) 1 (B) $-\frac{1}{2}$ (C) 2 (D) $\frac{1}{2}$ (E) $\infty$.

4. $\lim_{x\to 8} \frac{\sqrt{3x+1}-5}{x-8}$ = ? (A) 1 (B) $-\frac{1}{10}$ (C) $\frac{1}{10}$ (D) $\frac{1}{5}$ (E) $\frac{3}{10}$.

5. Find the interval of convergence (收斂區間) of the series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n}$ .
(A) (–1, 1) (B) (–1, 3) (C) (1, 2) (D) (1, 3) (E) $(-\infty, \infty)$ .

6. Use a linear approximation to estimate $\sqrt[3]{124.7}$ . (A) 4.993 (B) 4.994 (C) 4.995 (D) 4.996 (E) 4.997 .

7. Find the slant asymptote (**斜漸近線**) of the function $f(x) = \frac{3x^2+2}{x-3}$ . (A) $3x - y + 9 = 0$ (B) $x - 3y + 9 = 0$ (C) $3x + y - 9 = 0$ (D) $x + 3y - 9 = 0$ (E) No slant asymptote.

8. Find the directional derivative of the surface $z = f(x, y) = e^{x+y} - x^2 - y^2$ at the point $P(0, 0, 1)$ in the direction of $\hat{u} = [\cos(\frac{\pi}{6})]\hat{i} + [\sin(\frac{\pi}{6})]\hat{j}$ . (A) $\frac{1-\sqrt{3}}{2}$ (B) $\frac{1+\sqrt{3}}{2}$ (C) $\frac{1-2\sqrt{3}}{2}$ (D) $\frac{1+2\sqrt{3}}{2}$ (E) $\frac{1-\sqrt{3}}{4}$ .

9. Find the maximum directional derivative of the surface in Problem 8. (A) $\frac{1+\sqrt{3}}{4}$ (B) $\frac{1-\sqrt{3}}{4}$ (C) 1 (D) $\sqrt{2}$ (E) $\sqrt{3}$ .

10. $\int_0^2 \int_{\frac{y}{2}}^{2\sqrt{y}} dxdy$ = ? (A) $\frac{2\sqrt{2}}{3} + \frac{1}{2}$ (B) $\frac{4\sqrt{2}}{3} + \frac{1}{2}$ (C) $\frac{8\sqrt{2}}{3} + \frac{1}{2}$ (D) $\frac{4\sqrt{2}}{3} - 1$ (E) $\frac{8\sqrt{2}}{3} - 1$ .

11. $\int_0^{\frac{\sqrt{\pi}}{2}} \int_y^{\frac{\sqrt{\pi}}{2}} \sec^2(x^2)\, dxdy$ = ? (A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1 .

12. Find the area of the region bounded by the following lines: $x - y = 0$, $x - y = 2$ ; $2x + 3y = -1$, $2x + 3y = 3$ ; (A) $\frac{8}{5}$ (B) $\frac{8}{3}$ (C) $\frac{16}{5}$ (D) $\frac{16}{3}$ (E) $\frac{32}{5}$ .



背面尚有試題

13. Find the surface area of the surface : $z = f(x,y) = 16 - \frac{x^2}{2} - \frac{y^2}{2}$ that lies above the circle: $x^2 + y^2 \leqq 3$ . (A) $\frac{16}{3}\pi$ (B) $\frac{14}{3}\pi$ (C) $\frac{8}{3}\pi$ (D) $\frac{7}{6}\pi$ (E) $\frac{5}{3}\pi$

(Hint: surface area $S = \iint \sqrt{1 + f_x^2 + f_y^2}\ dxdy$ ) .

14. $\int_{-\infty}^{\infty} \frac{dx}{x^2+4}$ = ? (A) $\frac{1}{2}\pi$ (B) $\frac{1}{3}\pi$ (C) $\frac{1}{4}\pi$ (D) $\frac{1}{5}\pi$ (E) $\infty$ .

15. $\int_0^{\sqrt{2}} \frac{x+1}{x^2+2} dx$ = ? (A) $\frac{\sqrt{2}}{2}\pi + \frac{1}{2}\ln 2$ (B) $\frac{\sqrt{2}}{4}\pi + \frac{1}{2}\ln 2$ (C) $\frac{\sqrt{2}}{8}\pi + \frac{3}{2}\ln 2$ (D) $\frac{\sqrt{2}}{4}\pi + \frac{3}{2}\ln 2$ (E) $\frac{\sqrt{2}}{8}\pi + \frac{1}{2}\ln 2$ .

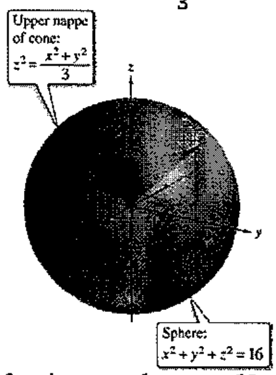
16. $\int_4^{49} \frac{dx}{\sqrt{\sqrt{x}+2}}$ = ? (A) $\frac{38}{3}$ (B) $\frac{43}{3}$ (C) $\frac{46}{3}$ (D) $\frac{52}{3}$ (E) $\frac{56}{3}$ .

17. Find the slope of the curve at the point $P(1,1)$ : $3x^2 + 4xy + y^2 - 2x = 0$. (A) $\frac{-2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{-8}{3}$ (D) $\frac{-4}{3}$ (E) $\frac{2}{3}$ .

18. Find the area of the surface generated by rotating the curve $\overrightarrow{r(t)} = x(t)\hat{i} + y(t)\hat{j}$ about the $x$-axis. The parameter equations, $x(t)$ and $y(t)$, are defined as $x(t) = t^2$, $y(t) = t$, $t \in [0, \sqrt{2}]$. (A) $\frac{13}{2}\pi$ (B) $\frac{13}{3}\pi$ (C) $\frac{13}{4}\pi$ (D) $\frac{13}{5}\pi$ (E) $\frac{13}{6}\pi$

(Hint: surface area $A = 2\pi \int_0^{\sqrt{2}} y(t)\sqrt{[x'(t)]^2 + [y'(t)]^2}\ dt$ ) .

19. Find the volume of the solid region bounded below by the upper nappe of the cone, $z^2 = \frac{x^2+y^2}{3}$ , and above by the sphere, $x^2 + y^2 + z^2 = 16$. (A) $\frac{32}{3}\pi$ (B) $\frac{32\sqrt{2}}{3}\pi$ (C) $\frac{64}{3}\pi$ (D) $\frac{64\sqrt{2}}{3}\pi$ (E) $\frac{128}{3}\pi$ .




20. Find the length of the curve $y = f(x) = \ln(\sec x)$ on the interval $x \in [0, \frac{\pi}{3}]$.

(A) $\ln(2+\sqrt{3})$ (B) $\ln(2-\sqrt{3})$ (C) $\ln(4+\sqrt{3})$ (D) $\ln(4-\sqrt{3})$ (E) $\ln(6-\sqrt{3})$ . (Hint: arc length $s = \int_0^{\frac{\pi}{3}} \sqrt{1 + [f'(x)]^2}\ dx$ ).